Choice under Risk and Uncertainty (Part II)

Week 11

M. Pelliccia

Recap- Exercise

Suppose that your utility function over lotteries that give you an amount x when a given event happens and y when it does not happen is

$$p\sqrt{x} + (1-p)\sqrt{y}$$

where p is the probability that the event happens and $(1-p)$ the probability of its complement.

Recap- Exercise (cont.)

a) If $p = 0.5$, calculate the utility of a lottery that gives you $\$10.000$ if the event happens and $\$100$ if it does not happen.
b) Calculate the certainty equivalent of this lottery.
c) If you were sure to receive $\$4.900$ what would your utility be?

Measures of Risk Aversion

In the previous lecture we have seen that:

- **Concave** vN-M Utility function implies **risk-aversion**.
- **Convex** vN-M Utility function implies **risk-lover**.
- **Linear** vN-M Utility function implies **risk-neutral**.

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Ho can we measure it analytically?

Measures of Risk Aversion (cont.)

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$A(c)$ can be constant (CARA), decreasing (DARA), or increasing (IARA).

Measures of Risk Aversion (cont.)

Arrow-Pratt-De Finetti Measure of Relative Risk Aversion, or **Relative Risk Aversion** (RRA) is defined as

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Again it can be CRRA, DRRA, or IRRA. It allows changes from risk aversion to risk seeking as c varies.

Measures of Risk Aversion (cont.)

- If an agent has increasing absolute risk aversion (IARA), then as wealth increases they will hold fewer pounds in risky assets.
- If an agent has constant absolute risk aversion (CARA), as wealth increases they will the same number of pounds in risky assets.
- If an agent has decreasing absolute risk aversion (DARA), then as wealth increases they will hold more pounds in risky assets.

Measures of Risk Aversion (cont.)

- If an agent has increasing relative risk aversion (IRRA), then as wealth increases they will hold a lower percentage of their wealth in risky assets.
- If an agent has constant relative risk aversion (CRRA), as wealth increases they will the same percentage of their wealth in risky assets.
- If an agent has decreasing relative risk aversion (DRRA), then as wealth increases they will hold a higher percentage of their wealth in risky assets.

Does the expected utility rule predict economic agents' behaviour? Does it provide us with a good description of individual behaviour under uncertainty?

Experimental results often suggest that individuals do not behave consistently with the expected utility rule.

How can we explain the behaviour of individuals who simultaneously buy insurance (say against losing their houses) and lottery tickets?

A way to explain it could be through a vN-M Utility function which is concave when it came to low payoffs and convex when it came to high ones.

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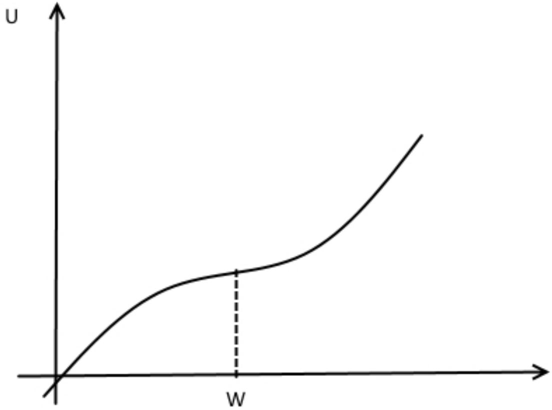

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Were such individuals to gain a lot of money, they would find themselves in the convex range of the function, and stop buying insurance.

Inflection point "moves" with wealth level. Thus, utility function which is **defined on changes of wealth** (relative terms) and not on its absolute levels!

Prospect Theory

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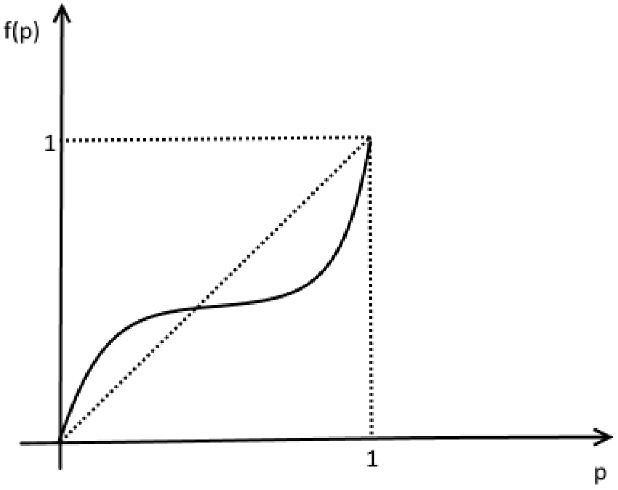
Individuals have *decision weights* described by a non-linear function $f(p)$ of probability p.

Prospect Theory (cont.)



Prospect Theory (cont.)

Assume that we are given the prospect of gaining x_i pounds with probability p_i, for $i = 1, ..., n$, and $\sum_i p_i = 1$. The vN-M expected utility theorem suggests that the decision maker maximises the formula,

$$p_1 u(x_1) + ... + p_n u(x_n)$$

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By contrast, prospect theory suggests that the decision maker maximises

$$f(p_1)u(x_1) + ... + f(p_n)u(x_n)$$

Prospect Theory (cont.)

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Kahneman and Tversky argue that people react differently to gains versus losses: they are averse to losses.

The *value function* is still defined on real numbers, but these numbers are changes from the reference point, rather than the absolute levels of wealth.

Other paradoxes

Here are some other examples:

- Framing effect
- Endowment effect
- Sunk costs
- Representativeness heuristic
- Anchoring
- Ellsberg paradox

Framing

Problem 1

A 65 years-old relative of yours suffers from a serious disease. It makes her life miserable, but does not pose an immediate risk to her life. She can go through an operation that, if successful, will cure her. However, the operation is risky; 30% of the patients undergoing it die. Would you recommend that she undergoes it?

Framing (cont.)

Problem 2

A 65 years-old relative of yours suffers from a serious disease. It makes her life miserable, but does not pose an immediate risk to her life. She can go through an operation that, if successful, will cure her. However, the operation is risky; 70% of the patients undergoing it survive. Would you recommend that she undergoes it?

Framing (cont.)

These two problems are identical.

If you consider two large groups of randomly selected individuals, and you give each group one of these problems, you are likely to find that a larger percentage of people recommend undergoing the operation in Problem 2 than in Problem 1.

Framing effect: The effect that the frame, or the representation, has on decision.

Other "real life" examples?

How to avoid it?

Endowment effect

You are given $\$1000$ for sure. Which of the following two options would you prefer?

1. To get an additional $\$500$ for sure.
2. To get another $\$1000$ with probability $1/2$, and otherwise nothing (and be left with the first $\$1000$).

Endowment effect (cont.)

You are given \$2000 for sure. Which of the following two options would you prefer?

1. To lose \$500 for sure.
2. To lose \$1000 with probability $1/2$, and otherwise to lose nothing.

Endowment effect (cont.)

Endowment effect: The tendency to value what we have more than what we do not yet have.

It can be viewed as a manifestation of a general principle called the *status quo bias*.

Endowment effect (cont.)

Consider the following example. We are trying to find out the value of a coffee mug for students. Here are three different ways to assess it:

- Ask the students how much they are willing to pay in order to get the mug.
- Tell the students that each of them is going to get a gift. It can be either the mug or a sum of money. Ask them what amount of money would make them indifferent between the mug and the money.
- Give each student the mug as a gift. Now ask them how much we will need to pay them in order to buy the mug from them.

Endowment effect (cont.)

Assume that the students' bundle is a pair (m, n), where m is the amount of money and n the number of coffee mugs in their possession. Let us say that they start with $(m, 0)$.

Question I looks for a price p such that

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Why do we observe the endowment effect? Is it rational?

Endowment effect (cont.)

Among the reasons,

- *Information:* When we owe something, we know it better than we don't.
- *Transaction costs:* If we had no status quo bias, we would be switching between different choices much more often than we do.
- *Habit formation:* We get used to certain products.

Sunk costs

You go to a movie. It was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

Sunk costs (cont.)

Your friend had a ticket to a movie. She couldn't make it, and gave you the ticket "instead of just throwing it away". The movie was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

Sunk costs (cont.)

The only difference between the two situations is that in problem I presumably you paid for the ticket and in problem II you did not.

Often people find it easier to walk out in the middle of a boring movie if they did not pay for it than if they did.

The amount of money you may have paid is a *sunk cost*, as it cannot be retrieved. It should be ignored.

Representativeness heuristic

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and she participated in anti-nuclear demonstrations.

Representativeness heuristic

Rank order the following eight description in terms of the probability (likelihood) that they describe Linda:

1. Linda is a teacher in an elementary school.
2. Linda works in a bookstore and takes yoga classes.
3. Linda is active in a feminist movement.
4. Linda is a psychiatric social worker.
5. Linda is a member of the League of Women Voters.
6. Linda is a bank teller.
7. Linda is an insurance salesperson.
8. Linda is a bank teller who is active in a feminist movement.

Representativeness heuristic (cont.)

- **Linda is a bank teller.**
- ...
- **Linda is a bank teller who is active in a feminist movement.**

The point is to see if you ranked 6 as less likely than 8.

It does not make sense to rank 6 less likely than 8 since 8 is fully contained in 6!

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Often, relying on logic and probability leaves us with no answer.The human mind has developed “heuristic” or methods to generate answers that are not guaranteed to provide a correct answer, but that do result in plausible answers most of the time.

Anchoring

A newly hired engineer for a computer firm in Melbourne, Australia, has four years of experience and good all-around qualifications. Do you think that her annual salary is above or below $65000? What is your estimate of her salary?

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Anchoring (cont.)

Typically, there is a statistically significant difference between the answer given in the two conditions; people give higher estimates if they are first asked about the higher value.

Anchoring effect: Irrelevant, or nearly irrelevant, information might have, above and beyond what can be reasonably justified.

Ellsberg paradox

Consider one urn containing 90 balls. Each ball can be Red, Blue, or Yellow. There are 30 Red balls (and thus 60 either Blue or Yellow balls). A ball is drawn at random from the urn.

You are offered choices between pairs of bets, where "betting on an event" implies winning $\$1,000$ if the event occurs, and nothing otherwise:

- Betting on the ball being Red
- Betting on the ball being Blue
- Betting on the ball being *not* Red
- Betting on the ball being *not* Blue

Ellsberg paradox (cont.)

The event "The ball is Red" has known probability of $1/3$, whereas for "The ball is Blue" the probability can be anywhere in the interval $[0, 2/3]$.

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The even "The ball is Not-Red" has known probability of $2/3$, whereas for "The ball is Not-Blue" the probability can be somewhere between $1/3$ and 1.

Ellsberg paradox (cont.)

Almost all participants prefer to bet on Red to betting on Blue and Not-Red to Not-Blue ($R \succ B$ and $\neg R \succ \neg B$).

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Many people seems to be *uncertainty averse*: other things being equal, they prefer known probabilities to unknown ones.

Ellsberg paradox (cont.)

In the words of Daniel Ellsberg,

Ambiguity *is a quality depending on the amount, type, reliability, and "unanimity" of information, and giving rise to one's "degree of confidence" in an estimate of relative likelihoods.*

Ellsberg paradox (cont.)

Over the last twenty years, several decision models that can accommodate ambiguity and ambiguity aversion (or appeal) have been axiomatized. There is an ever-growing collection of applications to contract theory, auctions, finance, macroeconomics, political economy, insurance and other areas of economic inquiry.

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Exercise

Assume that you are indifferent between getting \$700 and getting \$1000 with probability 80% (and otherwise nothing). Assume also that you are indifferent between getting \$300 and getting \$700 with probability 60% (and otherwise nothing). Consider lottery A, which gives you \$1000 with probability 2/3 (and otherwise nothing), and lottery B, which gives you a 50% – 50% bet between \$300 and \$700. If you follow von Neumann-Morgenstern's theory, you should:

a. Prefer A to B
b. Prefer B to A
c. Be indifferent between A and B
d. One cannot tell based on the data

Exercise

Suppose an individual has $\$100$ to invest. Two assets are available. One asset will yield a return of 10%, while the other risky asset will yield 0% with probability 0.5 and 21% with probability 0.5. Suppose the investor's utility function is given by $U(x) = \ln(x)$ where x is the wealth after investing (assume she is investing for just one period). How much will she invest in the risky asset?

Readings

- Chapter 16 Perloff
- Additional readings (links on Moodle)