Microeconomic Analysis

Consumer Choice

(Part II)

Outline

- Deriving Demand Curves
- Effect of Income on Demand
- Effect of Price Changes on Demand
- COLA

Demand Curve

- Demand curve summarises the relationship between quantity demanded and price, *ceteris paribus*

- Know that individual selects the greatest possible utility given the budget constraint
  - Can derive demand curve by varying one price and plotting the resulting quantity consumed

Budget Line, L

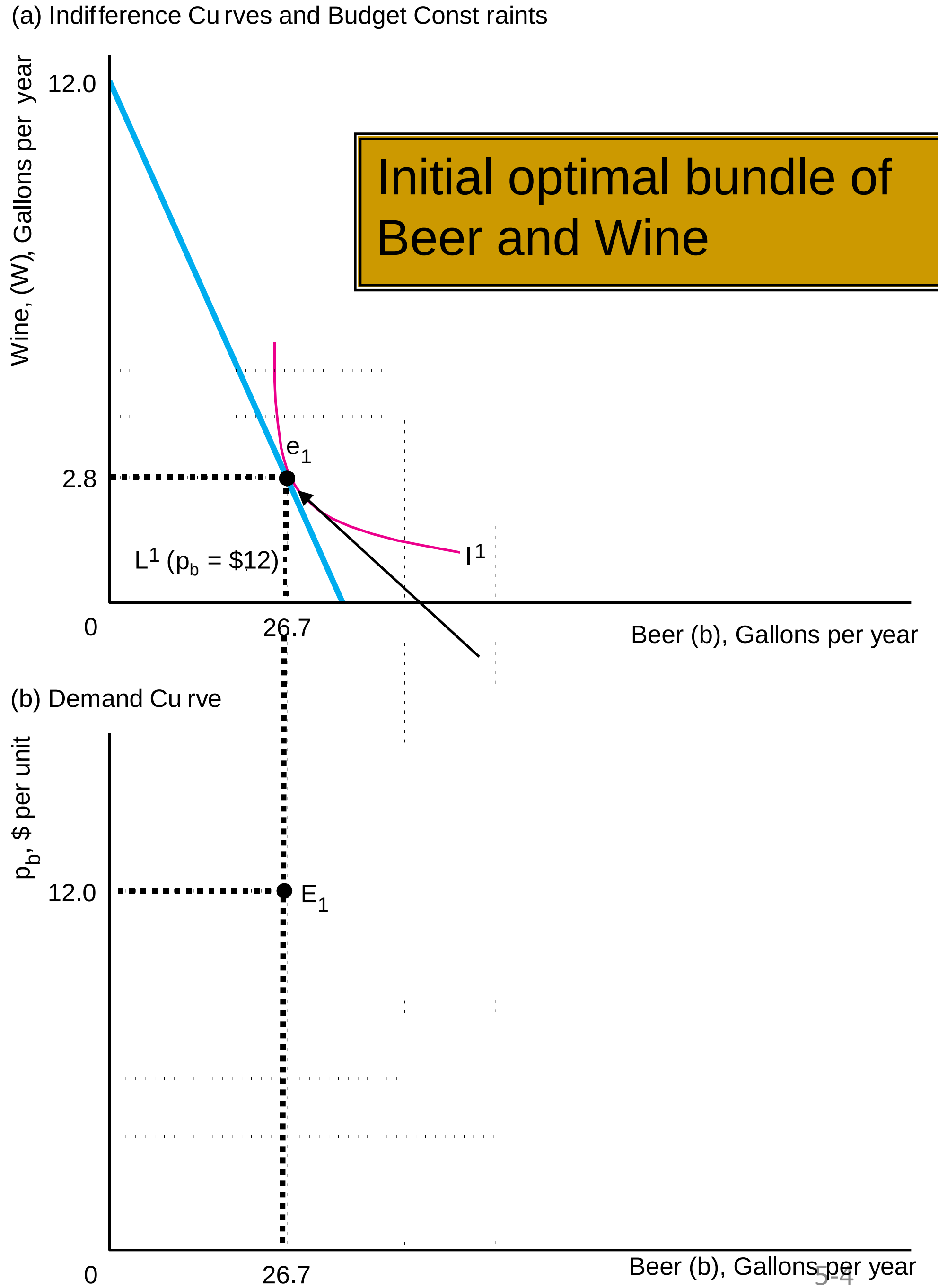
$$W = \frac{Y}{P_W} - \frac{P_B}{P_W} B$$

Initial Values

P_B = price of beer = $12

P_W = price of wine = $35

Y = Income = $419.



Budget Line, L

$$W = \frac{Y}{P_W} - \frac{P_B}{P_W}B$$

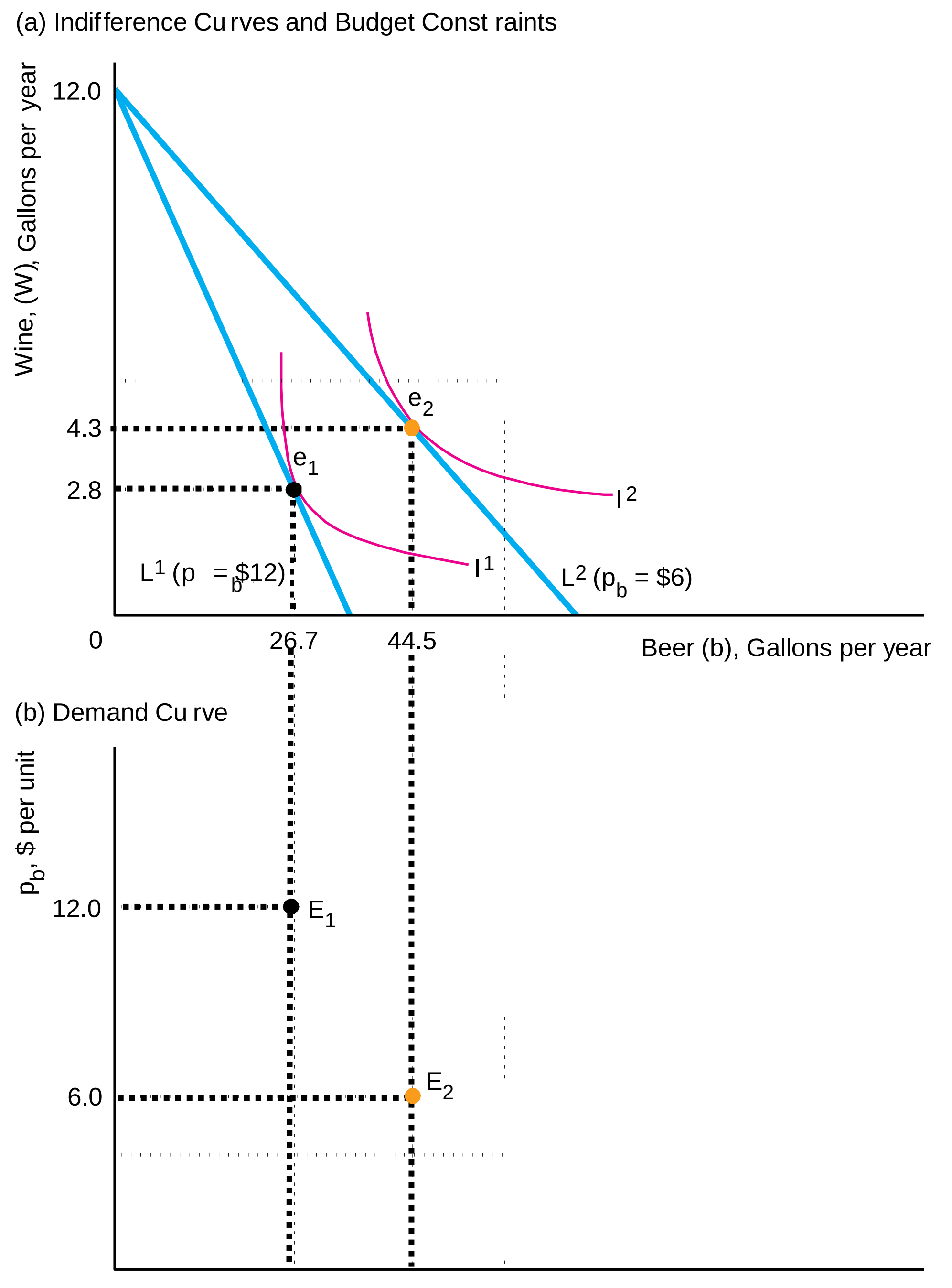
New Values

P_B = price of beer = $6

P_W = price of wine = $35

Y = Income = $419.

Price of Beer goes down!



$$W = \frac{Y}{P_W} - \frac{P_B}{P_W} B$$

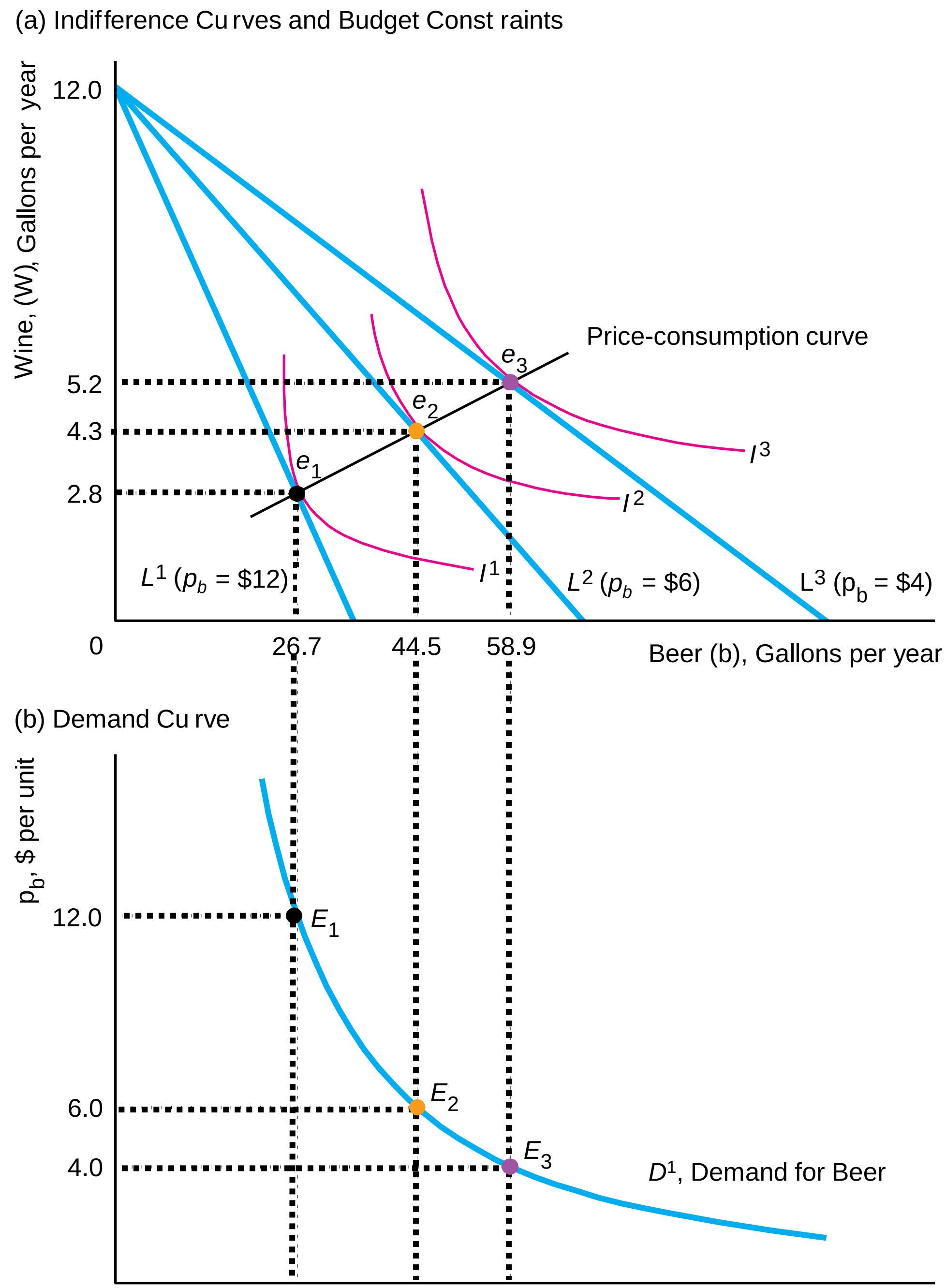
New Values

P_B = price of beer = $4

P_W = price of wine = $35

Y = Income = $419.

Price of Beer goes down again!

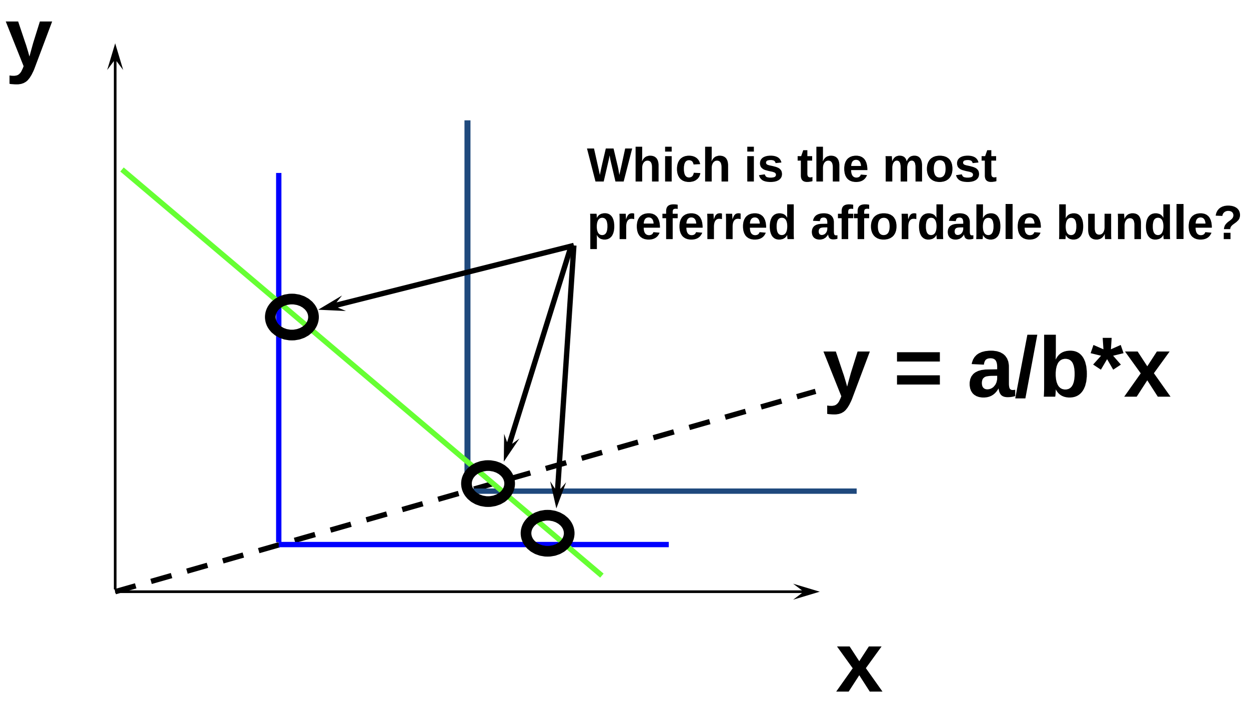


Derive the Walrasian demand function assuming $U(x,y)=Ax^{\alpha}y^{\beta}$ …

Perfect Complements

For (perfect) complements, the utility function is

$$U(x, y) = \min\{ax, by\}$$

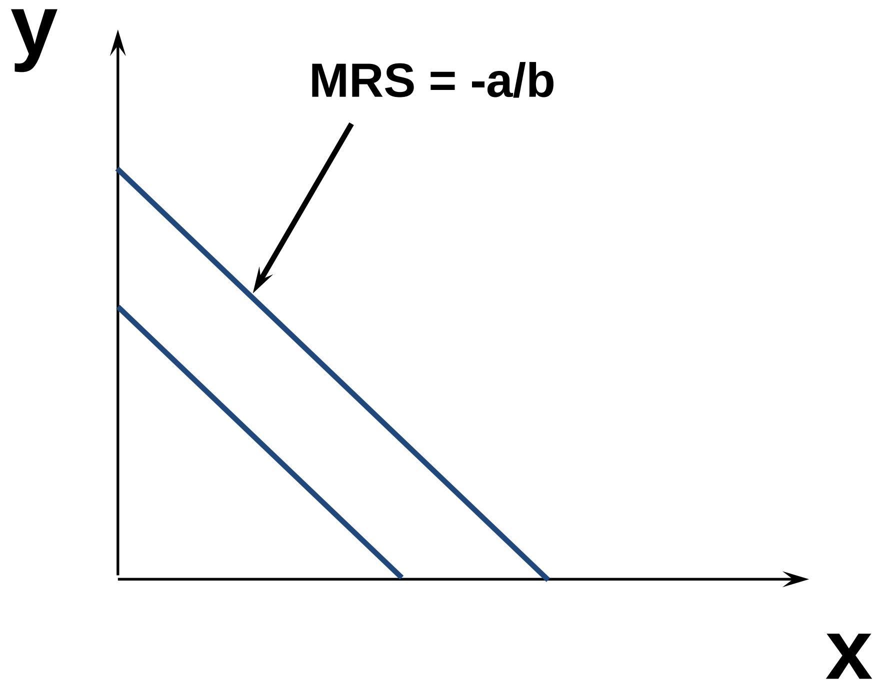


Deriving Demand Curves for (Perfect) Complements…

(Perfect) substitutes

For (perfect) substitutes the utility function is,

$$U(x, y) = ax + by$$



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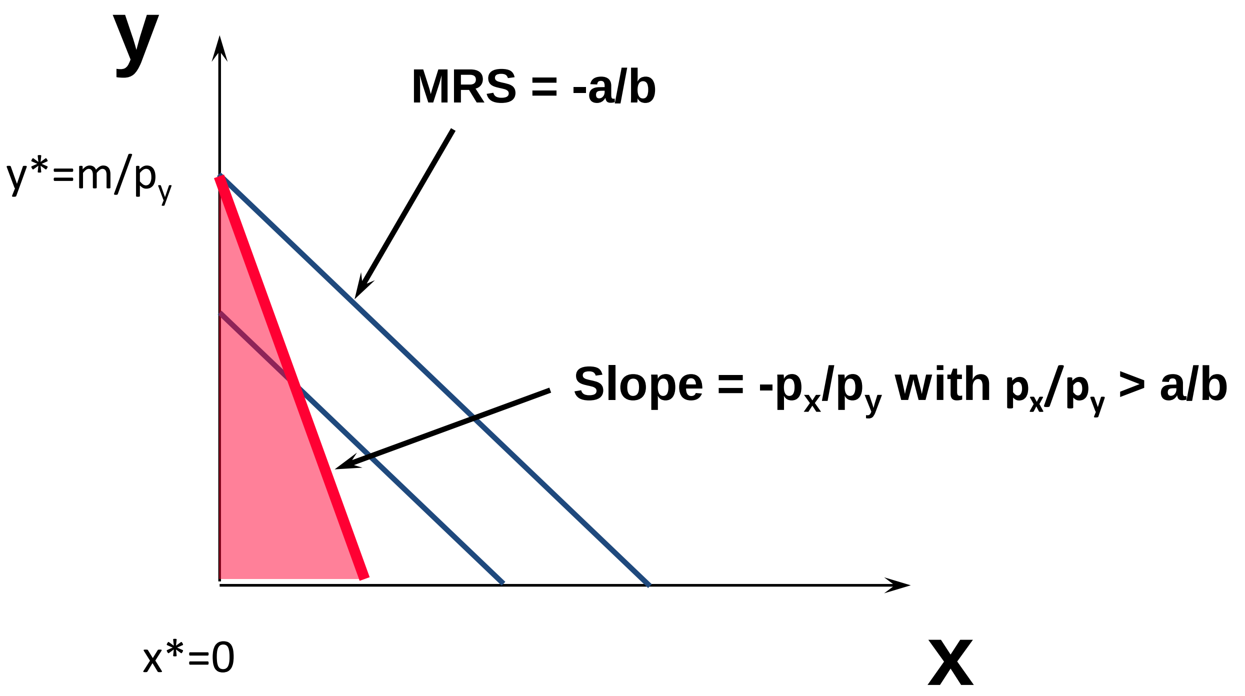

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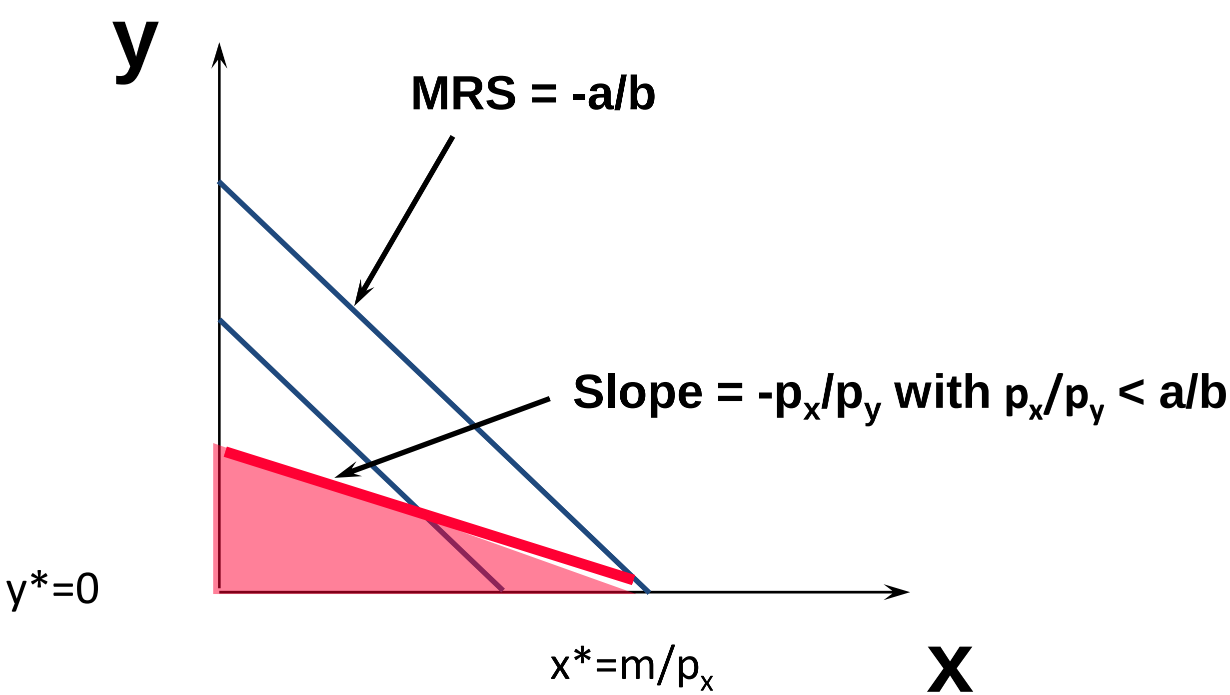


Deriving Demand Curves for (Perfect) Substitutes…

Changes in Income

- How an increase in income, when all prices are held constant causes a shift of the demand curve

- **Engel curve** - the relationship between the quantity demanded of a single good and income, holding prices constant

Budget Line, L

$$W = \frac{Y}{P_W} - \frac{P_B}{P_W} B$$

Initial Values

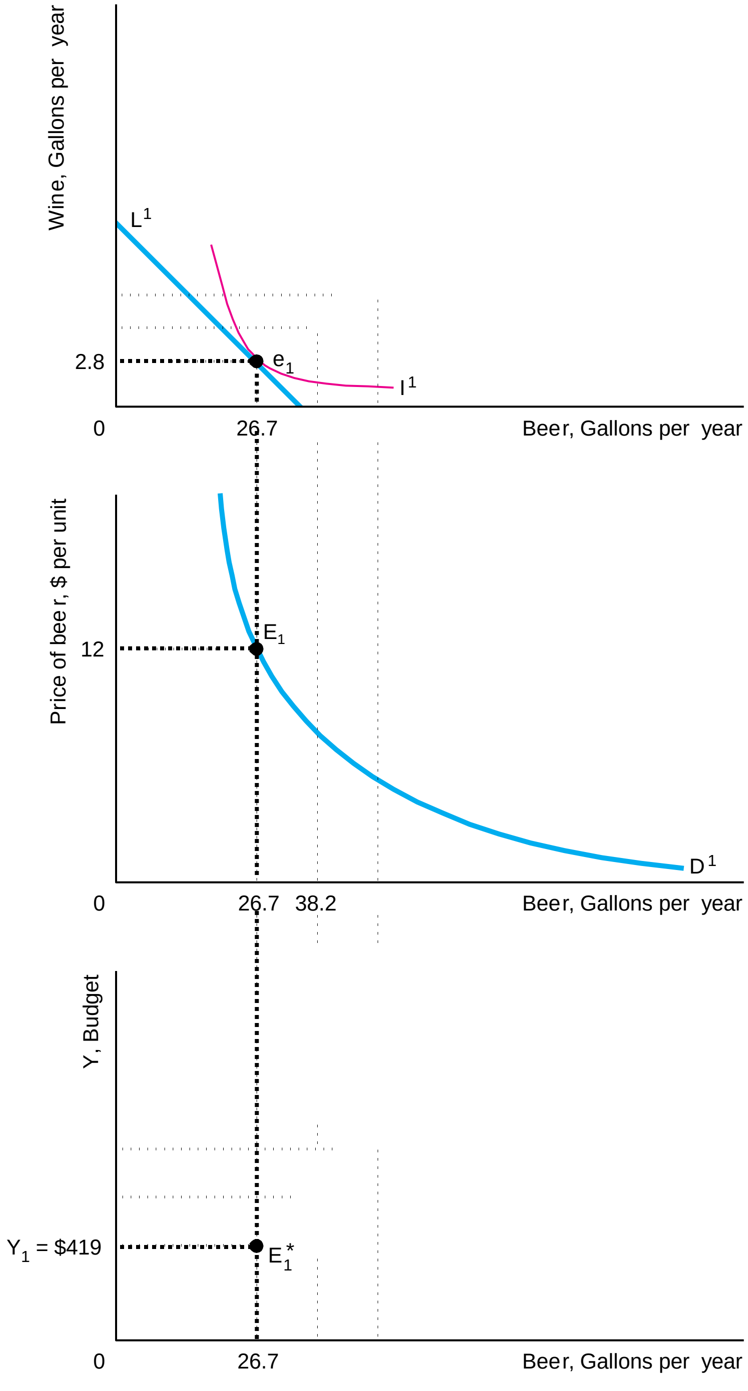
P_b = price of beer = $12

P_W = price of wine = $35

Y = Income = $419.

$628

Income goes up!



Budget Line, L

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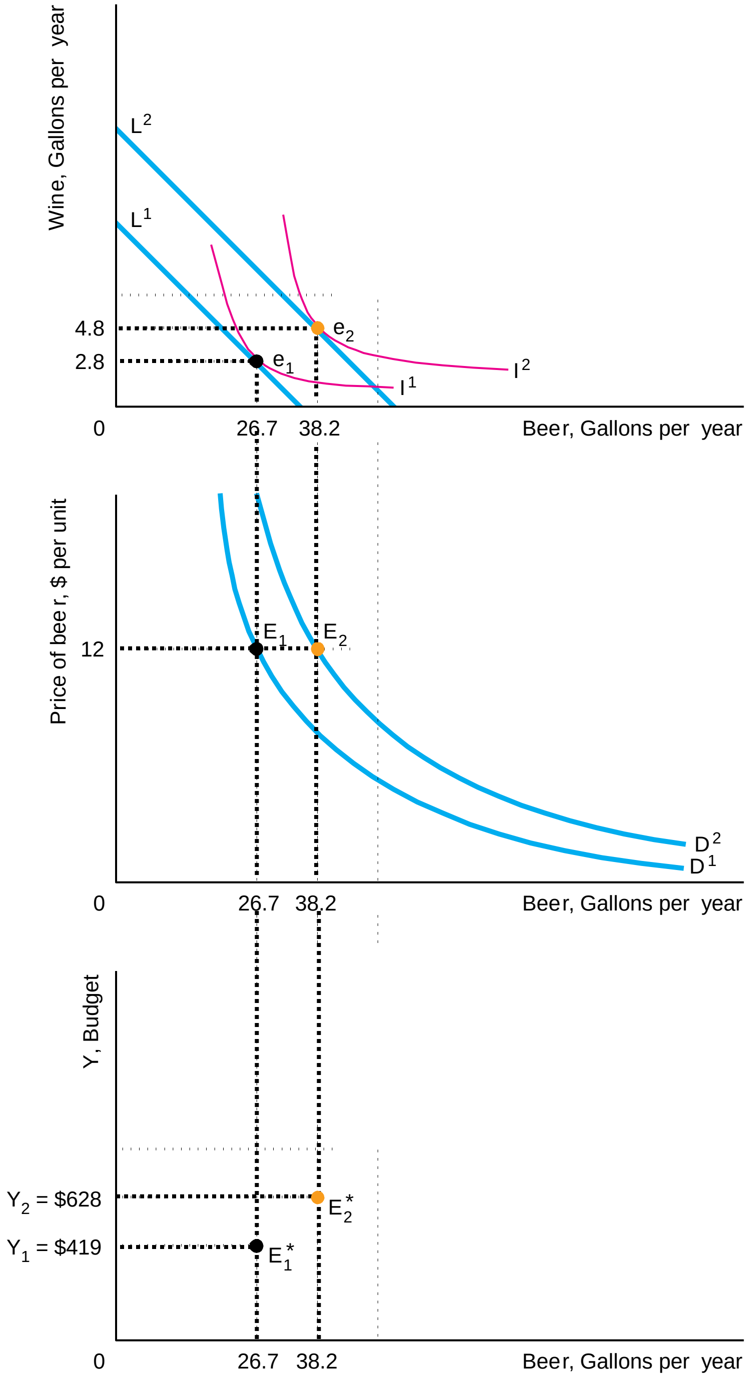


<u>Budget Line, L</u>

$$W = \frac{Y}{P_W} - \frac{P_B}{P_W}B$$

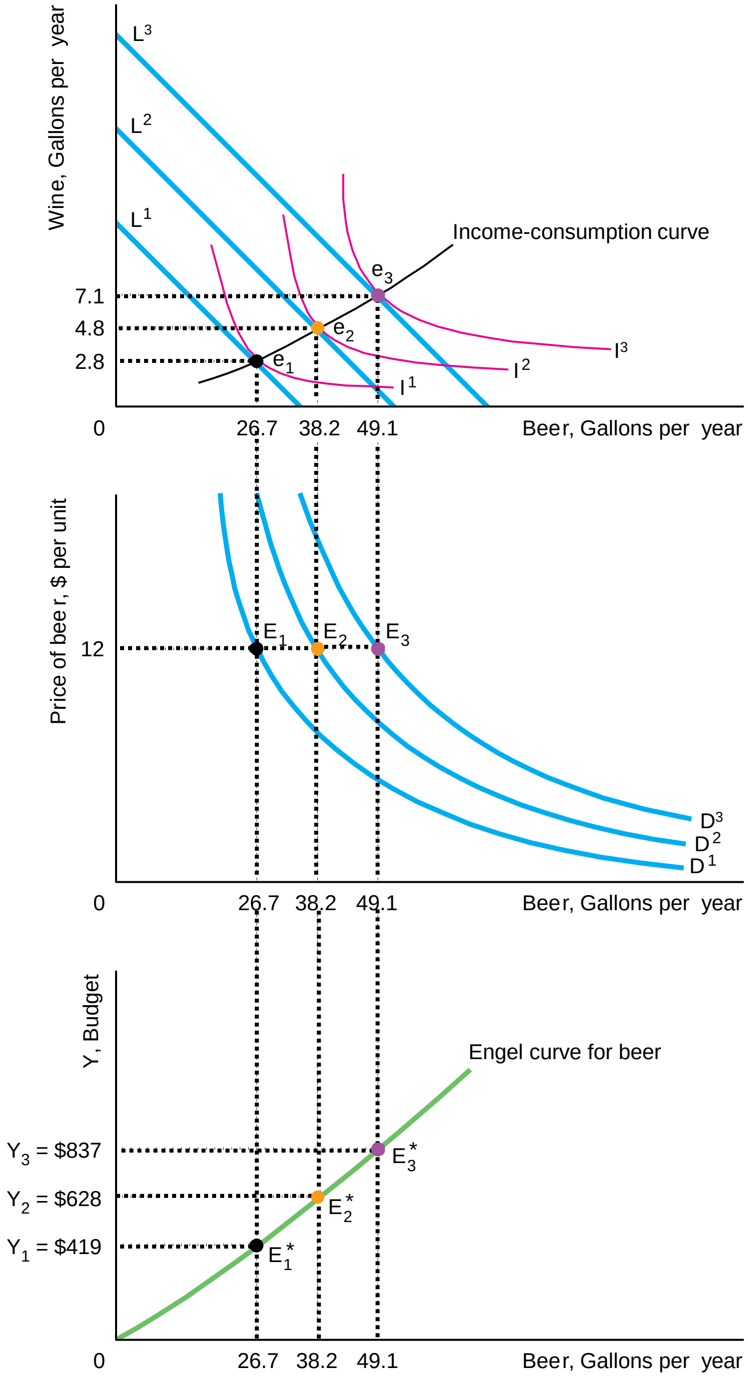
<u>Initial Values</u>

P_b = price of beer = \$12

P_W = price of wine = \$35

Y = Income = \$837.

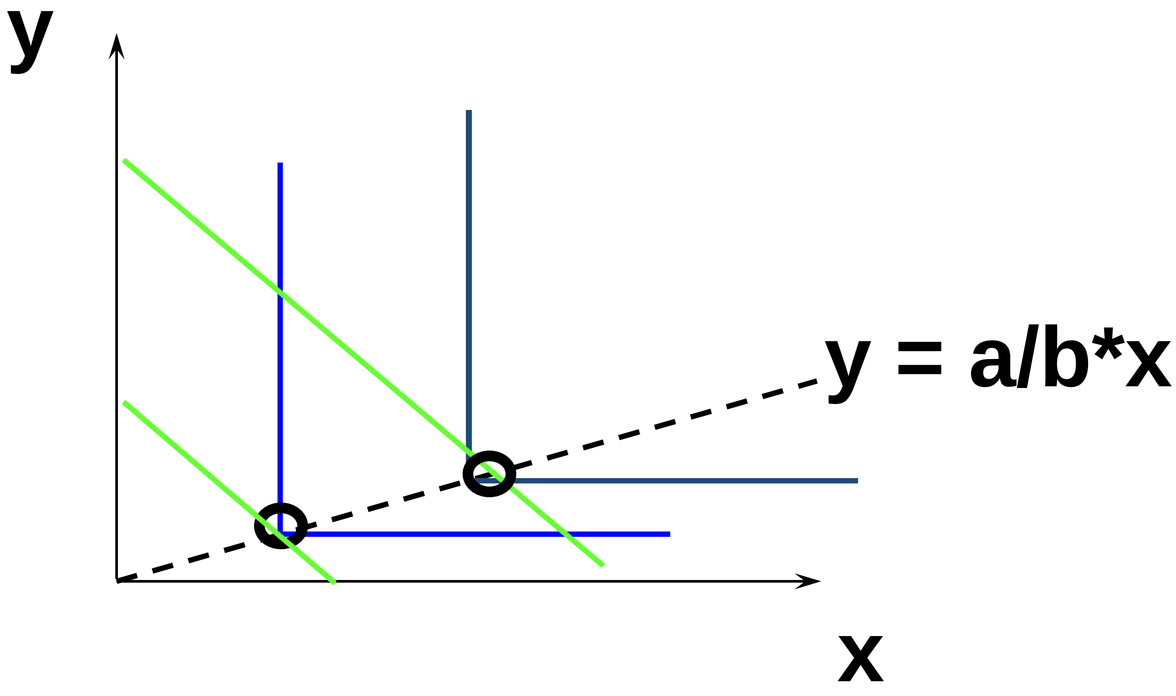
Income goes up again!



Changes in Income if (perfect) complements

For (perfect) complements, the utility function is

$$U(x,y)=\min\{ax,by\}$$



Changes in Income if (perfect) substitutes

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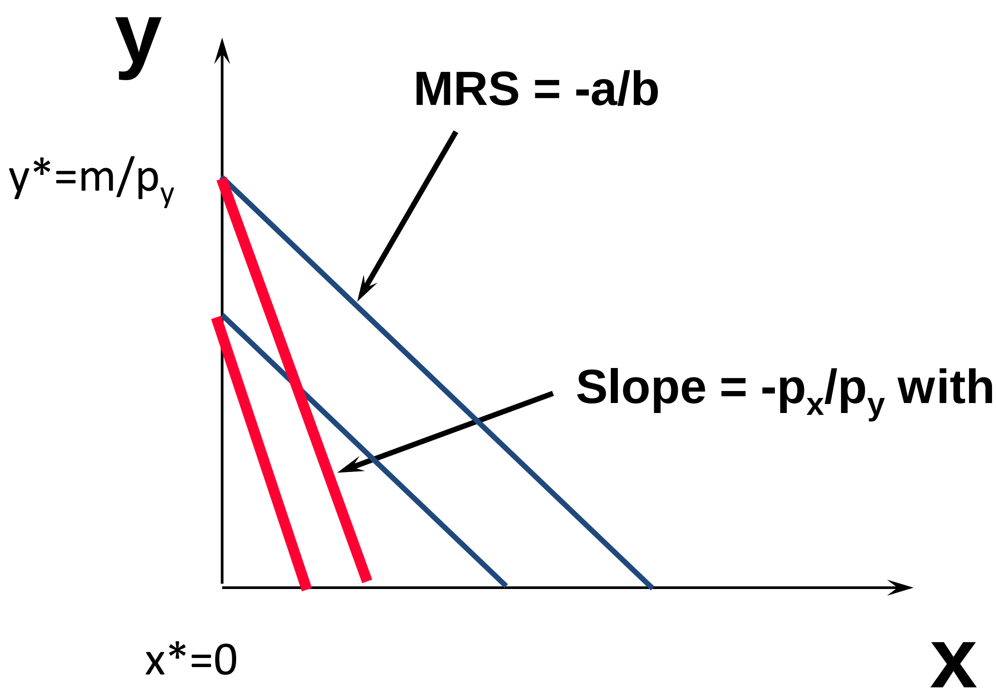


Income Elasticity

$$\xi = \frac{\%\Delta Q}{\%\Delta Y} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta Y}{Y}} = \frac{\Delta Q}{\Delta Y}\frac{Y}{Q}$$

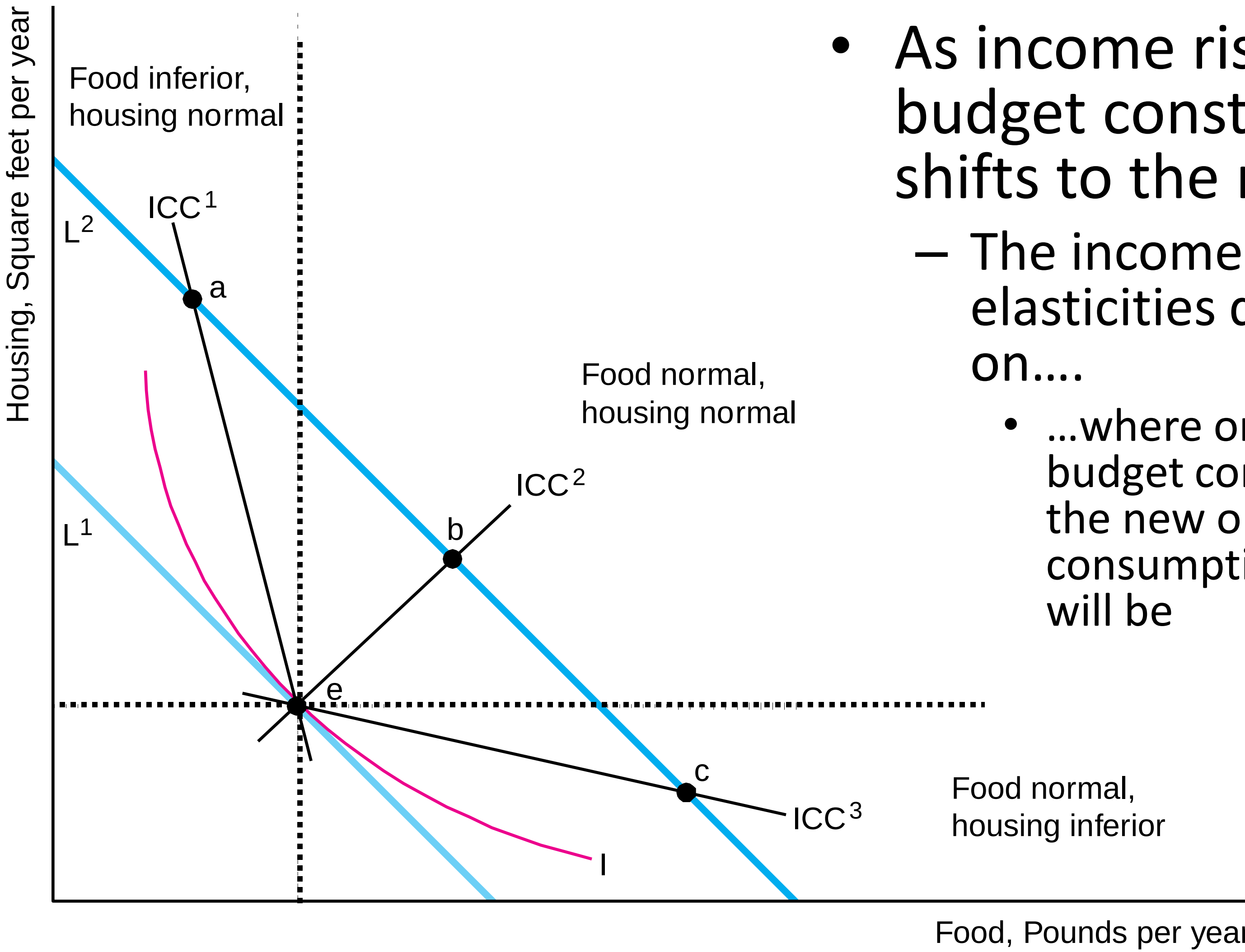
Example

If a 1% increase in income results in a 3% decrease in quantity demanded, the **income elasticity of demand** is x = -3%/1% = -3.

- **normal good -** a commodity of which as much or more is demanded as income rises
  - Positive income elasticity

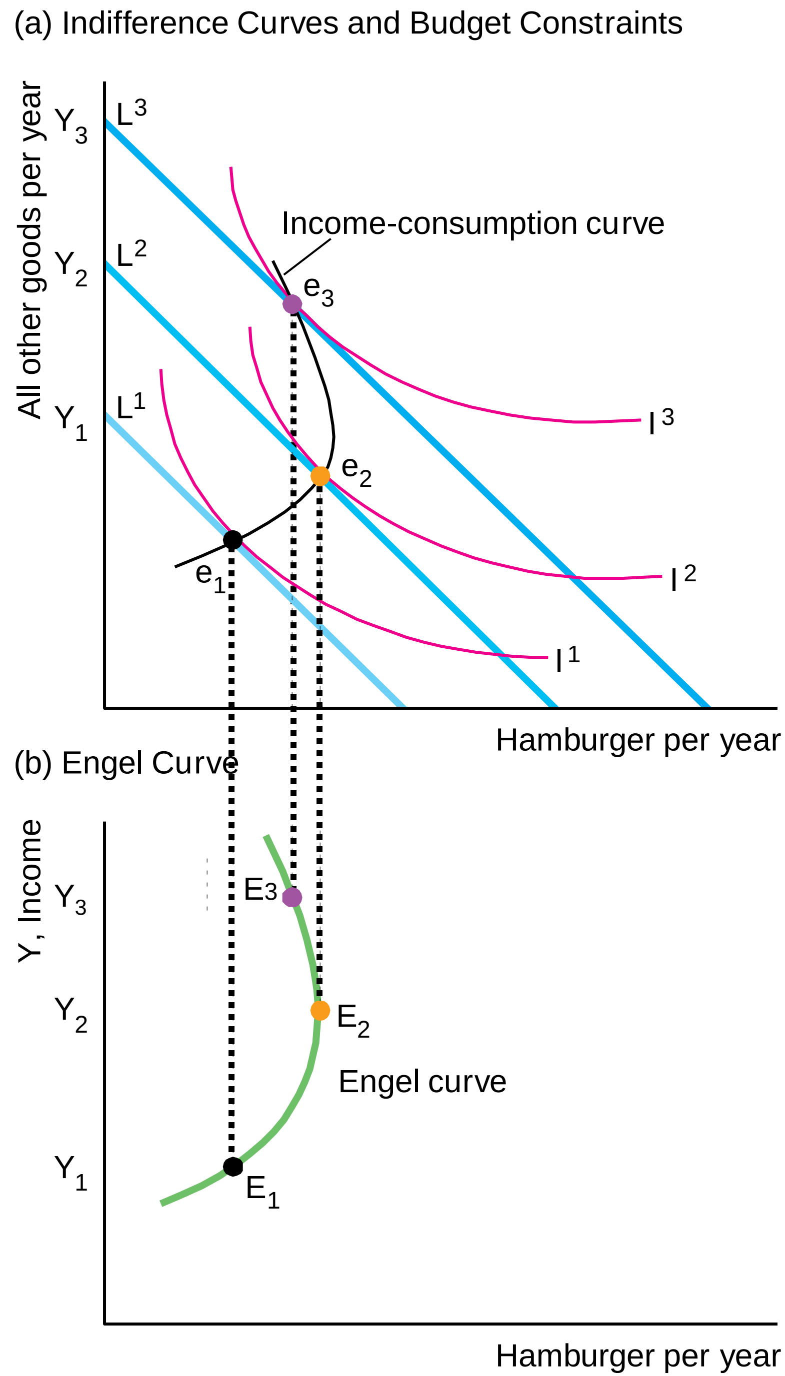
- **inferior good -** a commodity of which less is demanded as income rises
  - Negative income elasticity

Income-Consumption Curves and Income Elasticities



- As income rises the budget constraint shifts to the right.
  - The income elasticities depend on....
    - ...where on the new budget constraint the new optimal consumption bundle will be

- When Gail was poor and her income increased..
  - ...she bought more hamburger
- But as she became wealthier and her income rose...
  - ....she bought less hamburger and more steak.



Effects of a Price Change

- **substitution effect -** the change in the quantity of a good that a consumer demands when the good's price changes, holding other prices and the consumer's utility constant.

- **income effect -** the change in the quantity of a good a consumer demands because of a change in income, holding prices constant.

Substitution and Income Effects with a Normal Good

A decrease in the price of food has both an income effect and a substitution effect.

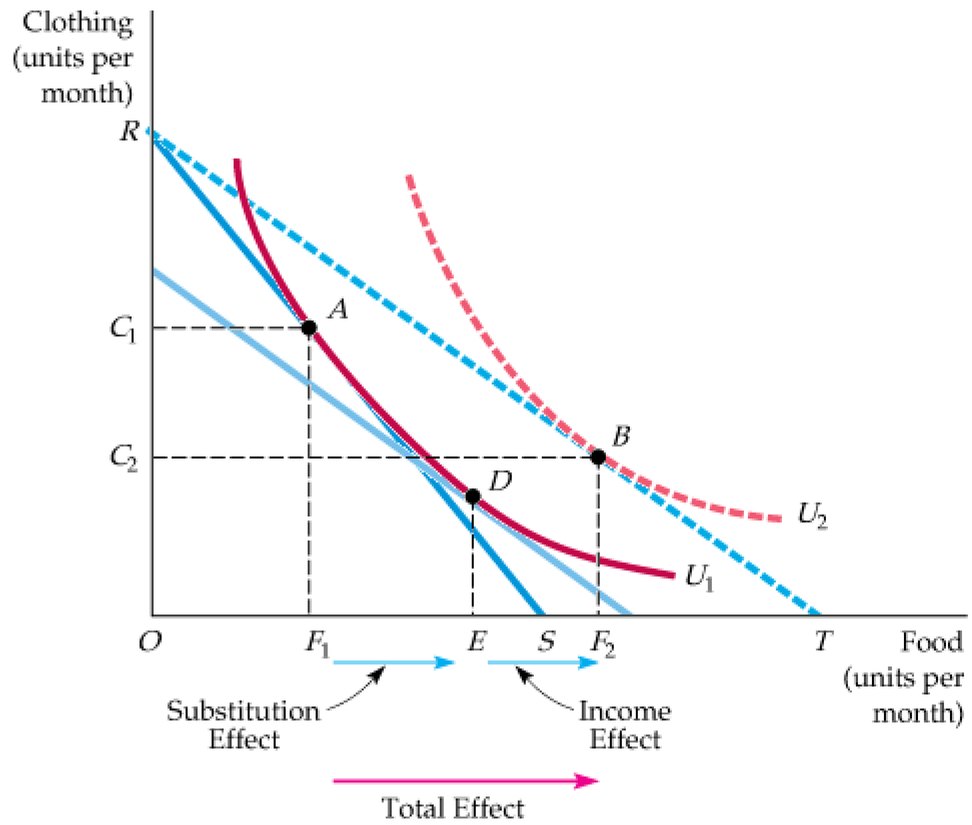
The consumer is initially at *A*, on budget line *RS*.

When the price of food falls, consumption increases by F_1F_2 as the consumer moves to *B*.

The substitution effect F_1E (associated with a move from *A* to *D*) changes the relative prices of food and clothing but keeps real income (satisfaction) constant.

The income effect EF_2 (associated with a move from *D* to *B*) keeps relative prices constant but increases purchasing power.

Food is a normal good because the income effect EF_2 is positive.



Decomposing the Effects of a Price Change

- While the direction of the substitution effect is always the same, (e.g., price decrease results in an increased quantity demanded and price increase results in a decreased quantity demanded), the income effect depends on whether the good is normal or inferior
- Recall that with an inferior good, the quantity demanded may actually decrease with higher income
- In this case, the income effect will be in the opposite direction to the substitution effect

Substitution and Income Effects with an Inferior Good

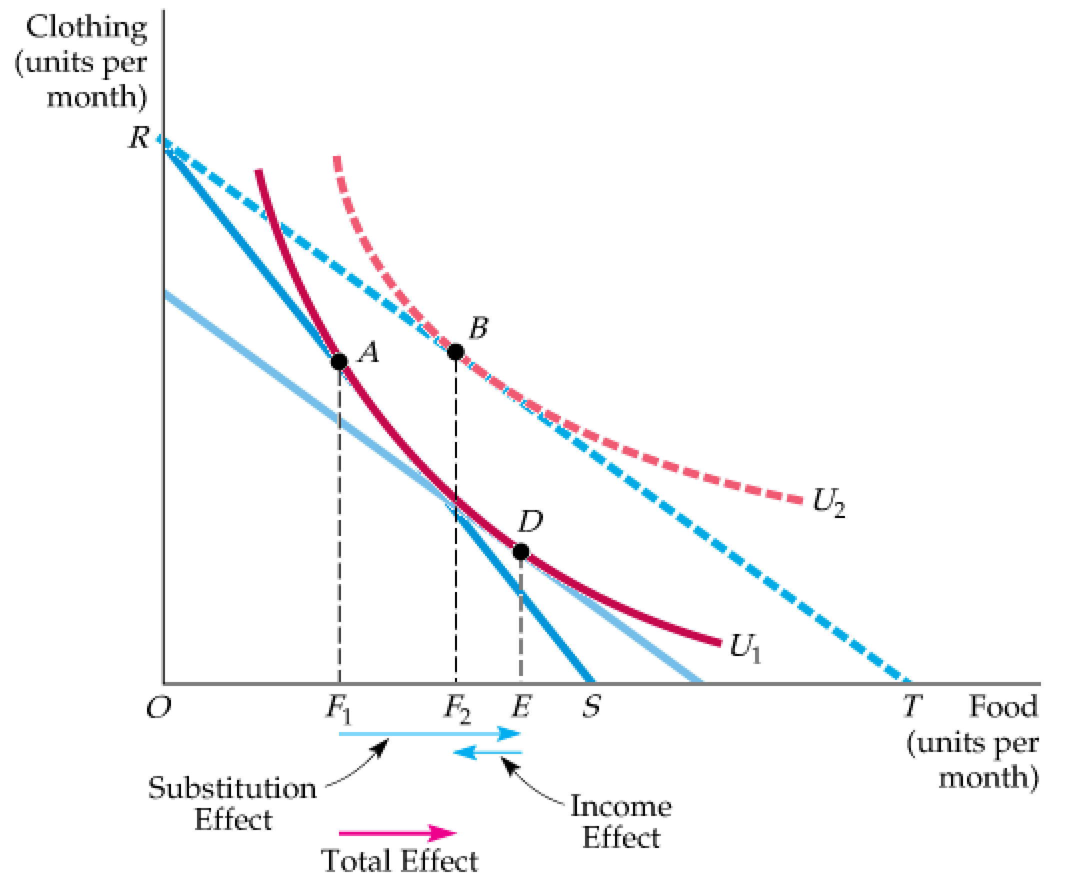
The consumer is initially at *A* on budget line *RS*.

With a decrease in the price of food, the consumer moves to *B*.

The resulting change in food purchased can be broken down into a substitution effect, F_1E (associated with a move from *A* to *D*), and an income effect, EF_2 (associated with a move from *D* to *B*).

In this case, food is an inferior good because the income effect is negative.

However, because the substitution effect exceeds the income effect, the decrease in the price of food leads to an increase in the quantity of food demanded.



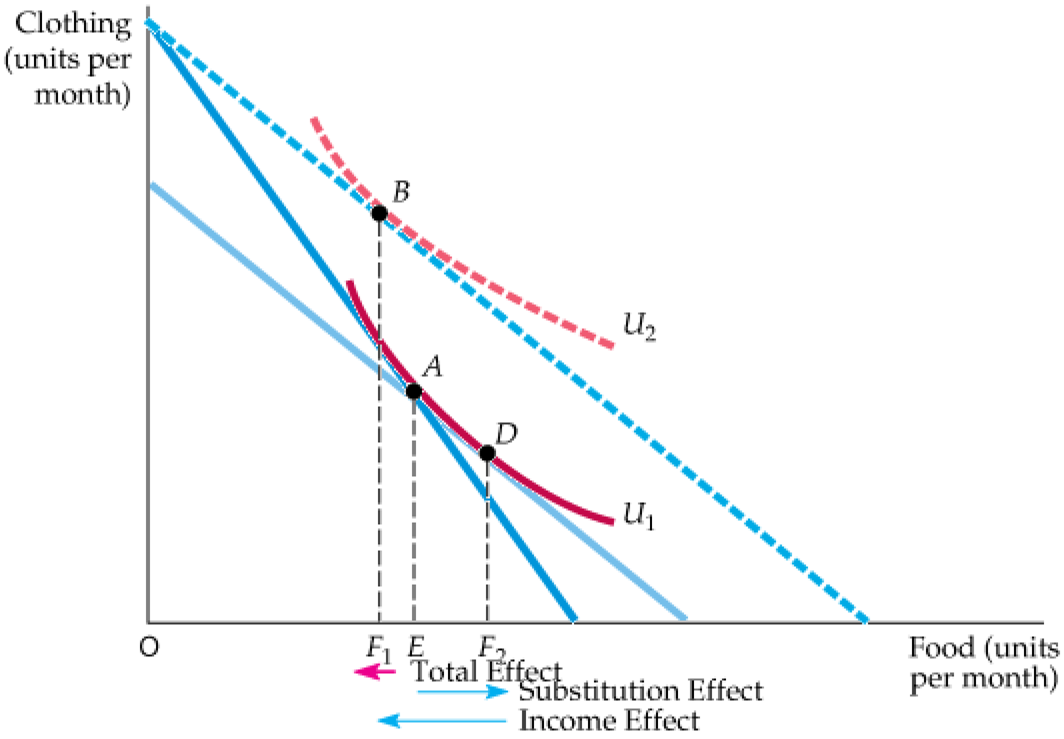
Substitution and Income Effects with an Inferior Good

- **Giffen good:** Good whose demand curve slopes upward because the (negative) income effect is larger than the substitution effect.

When food is an inferior good, and when the income effect is large enough to dominate the substitution effect, the demand curve will be upward-sloping.

The consumer is initially at point *A*, but, after the price of food falls, moves to *B* and consumes less food.

Because the income effect EF_2 is larger than the substitution effect F_1E, the decrease in the price of food leads to a lower quantity of food demanded.



Decomposing the Effects of a Price Change Mathematically (**<u>Slutsky Equation</u>**)

Suppose the demand for good x is $x(p_x, p_y, m)$. Let the Hicksian demand for good x be $h_x(p_x, p_y, \overline{U})$. Let the minimum level of income (or expenditure) necessary to attain utility $\overline{U}$ be $e(p_x, p_y, \overline{U})$. Then,

$$h_x(p_x, p_y, \overline{U}) \equiv x(p_x, p_y, e(p_x, p_y, \overline{U}))$$

$$\Rightarrow \frac{\partial h_x}{\partial p_x} = \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial m}\frac{\partial e}{\partial p_x}$$

$$p_x x + p_y y = m \Rightarrow \frac{\partial e}{\partial p_x} = x$$

$$\Rightarrow \frac{\partial h_x}{\partial p_x} = \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial m} x$$

$$\therefore \frac{\partial x}{\partial p_x} = \frac{\partial h_x}{\partial p_x} - x\frac{\partial x}{\partial m}$$

- **Normal good:** the overall effect of a price decrease is to increase quantity demanded
  - Substitute towards the relatively cheaper good while keeping utility constant
  - Extra income results in greater expenditure on the good
- **Inferior good:** overall effect of a price decrease is indeterminate
  - Substitution effect is negative as usual
  - However, the extra income results in decreased expenditure on the good

Inflation Indexes

- **Inflation** - the increase in the overall price level over time.
  - ***nominal price*** - the actual price of a good.
  - ***real price*** - the price adjusted for inflation.

- How do we adjust for inflation to calculate the real price?

Inflation Indexes (cont.)

- **Consumer Price Index (CPI)** – compares prices over time by indicating how much it costs to buy a given bundle of goods

  In nominal terms in 2008 the price of an hamburger is \$0.98 while in 1955 was \$0.15.

  – We can use the CPI to calculate the real price of a hamburger over time.

  – In terms of 2008 dollars, the real price of a hamburger in 1955 was:

$$\frac{\text{CPI for 2008}}{\text{CPI for 2005}} \times \text{price of a burger} = \frac{211.1}{26.8} \times 15 = 1.18$$

Calculating the CPI

- CPI is calculated by using baskets of goods and determining how much they cost
- Use base-year quantities

$$m_1 = p_{c1}c_1 + p_{f1}f_1$$

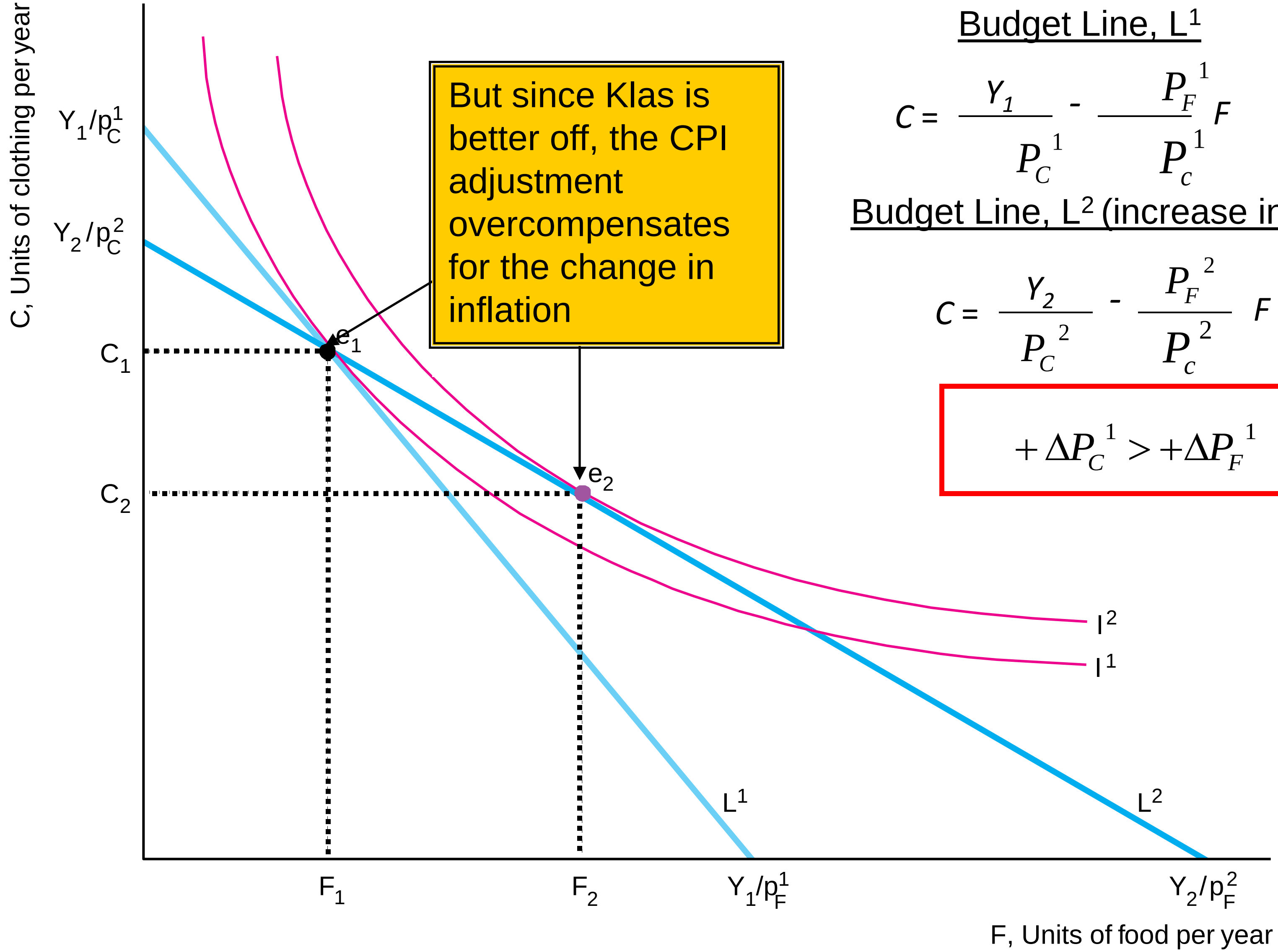
$$m_2 = p_{c2}c_1 + p_{f2}f_1$$

$$\Rightarrow CPI = \frac{m_2}{m_1} = \frac{p_{c2}c_1 + p_{f2}f_1}{p_{c1}c_1 + p_{f1}f_1}$$

$$\Rightarrow CPI = \frac{\frac{p_{c2}}{p_{c1}}p_{c1}c_1 + \frac{p_{f2}}{p_{f1}}p_{f1}f_1}{m_1}$$

$$\therefore CPI = \frac{p_{c2}}{p_{c1}}\theta_c + \frac{p_{f2}}{p_{f1}}\theta_f$$

The Consumer Price Index



<u>Budget Line, L^1</u>

$$C = \frac{Y_1}{P_C^{\,1}} - \frac{P_F^{\,1}}{P_c^{\,1}} F$$

<u>Budget Line, L^2 (increase in salary)</u>

$$C = \frac{Y_2}{P_C^{\,2}} - \frac{P_F^{\,2}}{P_c^{\,2}} F$$

$$+\Delta P_C^{\,1} > +\Delta P_F^{\,1}$$

Initial Consumption Bundle

- A true COLA would give just enough to leave utility unchanged

$$U(x,y)=20\sqrt{xy}\Rightarrow MRS=\frac{\partial U/\partial x}{\partial U/\partial y}=\frac{10x^{-0.5}y^{0.5}}{10x^{0.5}y^{-0.5}}=\frac{y}{x}$$

$$m_1=400,\ p_{x1}=1,\ p_{y1}=4,\ p_{x2}=2,\ p_{y2}=5$$

$$\text{YEAR ONE}:$$

$$MRS=MRT\Rightarrow\frac{y_1}{x_1}=\frac{1}{4}\Rightarrow x_1=4y_1$$

$$\text{Substitute into budget constraint } p_{x1}x_1+p_{y1}y_1=m_1$$

$$\Rightarrow 4y_1+4y_1=400$$

$$\therefore y_1=50,\ x_1=200,\ U(x_1,y_1)=2000$$

No Adjustment in Year Two

$$U(x,y) = 20\sqrt{xy} \Rightarrow MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{10x^{-0.5}y^{0.5}}{10x^{0.5}y^{-0.5}} = \frac{y}{x}$$

$$m_1 = 400,\ p_{x1} = 1,\ p_{y1} = 4,\ p_{x2} = 2,\ p_{y2} = 5$$

YEAR TWO WITHOUT ADJUSTMENT:

$$MRS = MRT \Rightarrow \frac{y_2}{x_2} = \frac{2}{5} \Rightarrow x_2 = 2.5y_2$$

Substitute into budget constraint $p_{x2}x_2 + p_{y2}y_2 = m_2$

$$\Rightarrow 5y_2 + 5y_2 = 400$$

$$\therefore y_2 = 40,\ x_2 = 100,\ U(x_2, y_2) = 1265$$

CPI Adjustment in Year Two

$$U(x,y) = 20\sqrt{xy} \Rightarrow MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{10x^{-0.5}y^{0.5}}{10x^{0.5}y^{-0.5}} = \frac{y}{x}$$

$$m_1 = 400, p_{x1} = 1, p_{y1} = 4, p_{x2} = 2, p_{y2} = 5$$

YEAR TWO WITH CPI ADJUSTMENT:

Budget constraint must have slope $-\frac{2}{5}$ and pass through point $(200,50)$

$$\Rightarrow Y_2 = 2 \cdot 200 + 5 \cdot 50 = 650$$

Consumer should be given income of £650 in year two in order for year one's consumption bundle to be affordable

$$\Rightarrow 2x_2 + 2x_2 = 650$$

$$\Rightarrow 4x_2 = 650$$

$$\therefore x_2 = 162.5,\ y_2 = 65, U(x_2, y_2) = 2055$$

True COLA in Year Two

$$U(x,y)=20\sqrt{xy}\Rightarrow MRS=\frac{\partial U/\partial x}{\partial U/\partial y}=\frac{10x^{-0.5}y^{0.5}}{10x^{0.5}y^{-0.5}}=\frac{y}{x}$$

$m_1=400,\ p_{x1}=1,\ p_{y1}=4,\ p_{x2}=2,\ p_{y2}=5$

YEAR TWO WITH TRUE COLA:

Want m_2 such that $20\sqrt{x_2y_2}=2000$

$$y_2=\frac{2}{5}x_2$$

$$\Rightarrow 20\sqrt{\frac{2}{5}x_2x_2}=2000$$

$$\Rightarrow \sqrt{\frac{2}{5}x_2x_2}=100$$

$$\Rightarrow \frac{2}{5}x_2x_2=100^2$$

$$\Rightarrow x_2^2=25000$$

$$\Rightarrow x_2=\sqrt{25000}$$

$\therefore x_2=158.11,\ y_2=63.44,\ U(x_2,y_2)=2000$

Consumer should be given income of £633.42 in year two in order for year one's utility level to be reached