Microeconomic Analysis

Game Theory (Part II)

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SOAS, 2014

Outline

Part II:

- Sequential games
    - Entry game
    - Hold up problem
    - Stackelberg's duopoly game
- Repeated games: Intuition

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Sequential game

We define each possible sequence of actions as *terminal history* and the function that gives the player who moves at each point in each terminal history as *player function*. Therefore, a **sequential game** requires

- Players
- Terminal histories
- Player function
- Preferences for the players

Example 1: Entry game

A *incumbent* (monopolist) faces the possibility of entry by a *challenger* (firm). If the challenger enters, the incumbent may either *acquiesce* or *fight*. In such case,

- Players: $\{\text{Incumbent}, \text{Challenger}\}$.
- Terminal histories: (In, Acquiesce), (In, Fight), (Out, Acquiesce), (Out, Fight).
- Player function: The Challenger starts the game and the Incumbent moves at the history In.
- Preferences for the players: functions u_c and u_i such that $u_c(\text{In, Acquiesce}) = 2$, $u_c(\text{Out}) = 1$, $u_c(\text{In, Fight}) = 0$, and $u_i(\text{In, Acquiesce}) = 1$, $u_i(\text{Out}) = 2$, $u_i(\text{In, Fight}) = 0$.

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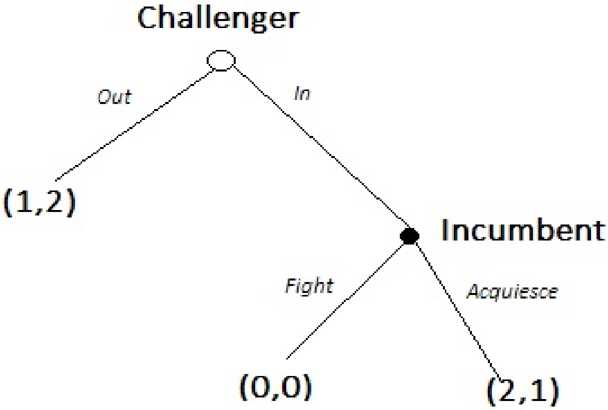
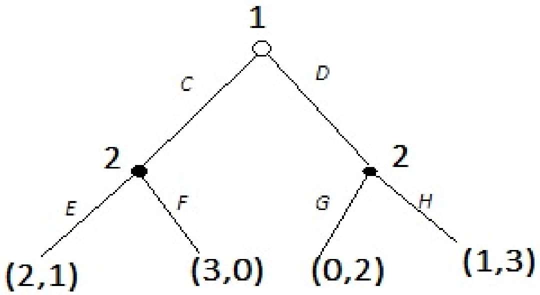

Challenger
Out
In
(1,2)
Incumbent
Fight
Acquiesce
(0,0)
(2,1)

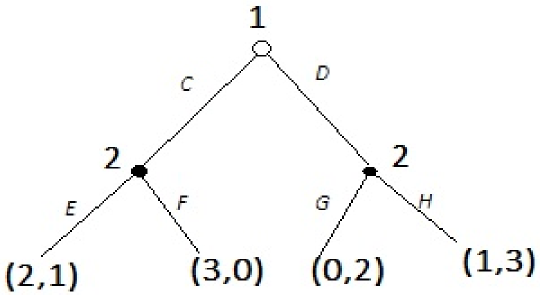
Definition

A **strategy** of player *i* in an extensive game with perfect information is a function that assigns to each history *h* after which it is player *i*'s turn to move an action in the set of actions available after *h*.



For player 1 the strategies are: *C* and *D*. For player 2: *E* if *C*, *F* if *C*, *G* if *D*, and *H* if *D*.

In every game, a player's strategy profile provides her *plan of action*: the actions she intends to take, *whatever* the other players do.



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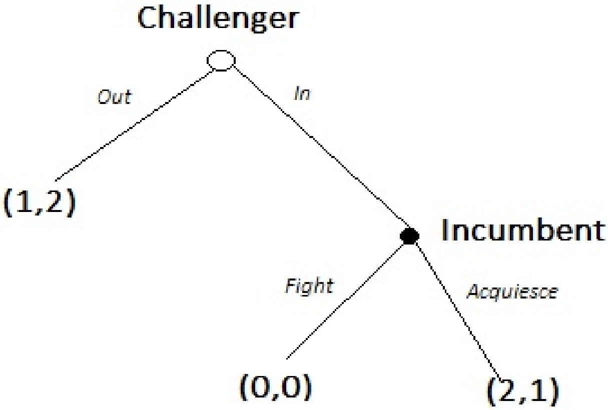
Solving the game by **backward induction** captures credibility!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It is not credible that the incumbent will fight. Hence, the entrant should come in (and the incumbent will not fight in response).

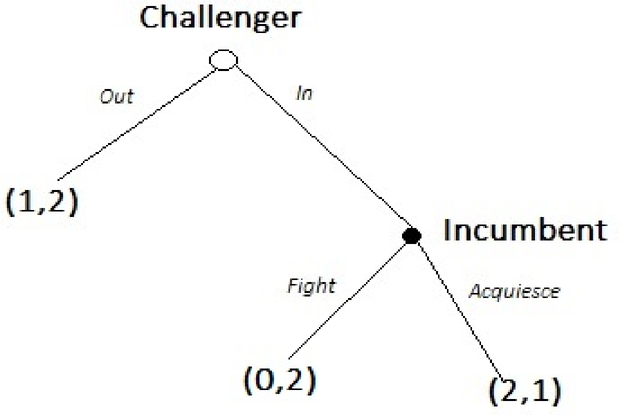
The problem of the incumbent is that he cannot pre-commit himself to fighting if the other firm enters.

Suppose that the incumbent can purchase some extra production capacity that will allow him to produce more output at his current marginal cost.

If he remains a monopolist, he does not use the extra capacity.

If the other firm enters, then the incumbent uses the extra capacity and this lowers the cost of fighting.

Suppose that the payoffs are as follows:



Now the threat of fighting is credible!$\Rightarrow$ The monopolist will stay a monopolist and never uses his extra capacity. Despite this, it is worthwhile to invest in the extra capacity in order to make credible the threat of fighting.

The monopolist has signalled to the potential entrant that he will be able to successfully defend his market.

Example 2: Hold up problem

You hire a contractor to build a warehouse. After the construction is almost done, you ask the contractor to change the colour of the paint (involving a trivial expense).

The contractor asks for a large price increase.

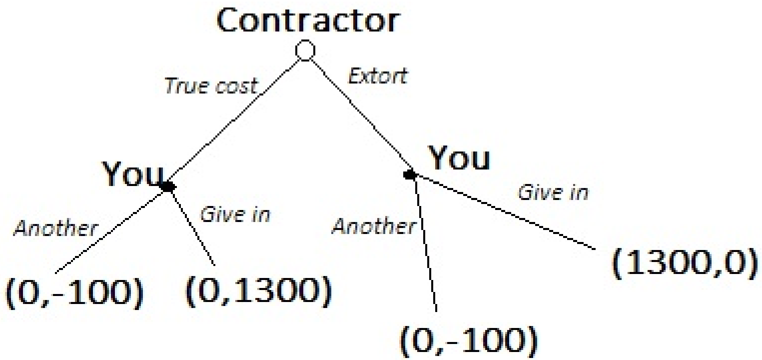
You pay.

Why?!

Example 2: Hold up problem

Assume that the new colour values 1500 to you. The cost of changing paint for the contractor is 200. If you do not give in a look for another decorator, you pay the true cost (200), but it takes time that you worth 1400. Suppose that the contractor can either extort (1500), or charge you the true cost (200). You can wither give in, or look for another decorator.

Example 2: Hold up problem



Example 2: Hold up problem

The optimal strategy for you is always to give in, and for the contractor to extort the large payment.

This is the reason why it could be optimal to write contracts that specify every possible contingency. But it might be too costly or impossible. Other solution?

Reputation obtained through repeated interaction.

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Stackelberg's oligopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially (*Leader* and *Follower*)?

Suppose $c_1 = c_2 = c$ and that firm 1 is the leader and firm 2 the follower. We may use *backward induction* to find the **subgame perfect equilibrium**.

- First, for *any* output q_1, we find the output q_2 that maximises her profit. Next, we find the output q_1 that maximises her profit, *given the strategy* of the follower.

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Stackelberg's oligopoly model (cont.)

Follower:

- We found that under the assumption of the Cournot's duopoly game Firm 2 has a unique best reply to each q_1, given by
$$q_2 = \frac{1}{2}(a - c - q_1)$$
(recall that $c_1 = c_2 = c$)

Stackelberg's oligopoly model (cont.)

Leader:

- The Leader's strategy is the q_1 which maximises $\pi_1 = (a - q_1 - q_2 - c)q_1$ subject to $q_2 = \frac{1}{2}(a - c - q_1)$. Thus,

$$\pi_1 = (a - q_1 - (\frac{1}{2}(a - c - q_1)) - c)q_1 = \frac{1}{2}q_1(a - q_1 - c)$$

which gives the maximiser

$$q_1^* = \frac{1}{2}(a - c)$$

Therefore, the unique (subgame perfect) equilibrium for this Stackleberg's duopoly game is $q_1^* = \frac{1}{2}(a - c)$ and $q_2^* = \frac{1}{4}(a - c)$.

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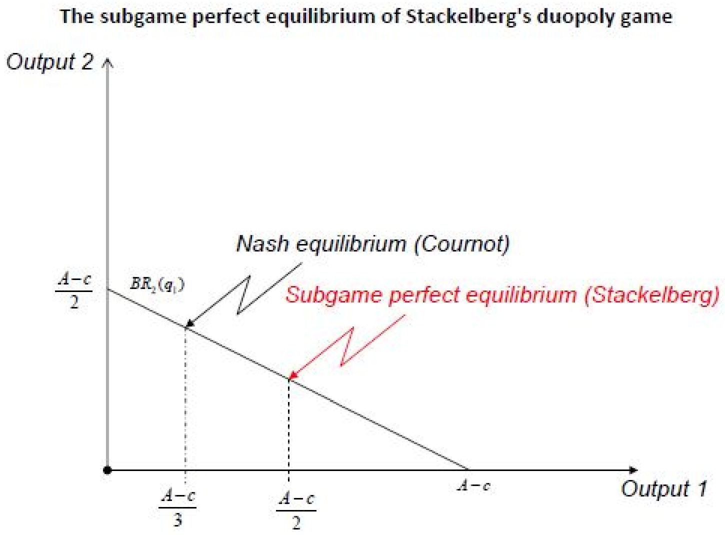

Exercise: Firm-Union bargaining

Suppose that a firm's output Q is defined as follows:

$$Q(L) = \begin{cases} L(100 - L) & \text{if } L \leq 50 \\ 2500 & \text{if } L > 50 \end{cases}$$

The price per unit is $p = 1$. A union (U) represents the workers presents a wage demand $w \in \mathbb{R}^+$, which the firm (F) can either accept (A) or reject (R). If F accepts, then she chooses L^* to employ. If F rejects, then $Q = 0$ and thus $L = 0$. The firm preferences are represented by the profit while the U's preferences by wL.

Exercise: Firm-Union bargaining

- Formulate the situation as an extensive game with perfect information.

  The *Players* are F and U. The *Terminal Histories* are $(w, A->L)$ and (w, R). The *Player Function* implies that U plays at the beginning and F chooses after U proposed w. The *Preferences* are represented by the profit for F and by wL for U.

Exercise: Firm-Union bargaining

- Find the SPNE of the game.

Solve by backward induction.

If $L \leq 50$ and U proposed w, the profit of F is $L(100 - L) - wL$ which implies $L^* = 50 - w/2$. In particular, this means that any $w \geq 100$, $L = 0$, or $Q = 0$. This also means that if U proposes any $w \geq 100$, she expects F to shut-down and thus $wL = 0$, while for any $0 < w < 100$, F produces using L^* and thus $wL > 0$.

This finally implies that maximising U's payoff function $wL = w(50 - w/2)$, we get $w^* = 50$, and thus a payoff of $50 \cdot 25 = 1250$. Consequently, for $w^* = 50$ and then $L^* = 25$, $\pi = 625$.

Summarising, the SPNE is $\{w^* = 50, (A- > L^* = 25)\}$.

Exercise: Firm-Union bargaining

- Is there an outcome of the game that both F and U prefer to any SPNE?

U and F would be better off if and only the following conditions are met,
If $L < 50$,

- $wL > 1250$ and $L(100 - L) - wL > 625$

or if $L \geq 50$

- $wL > 1250$ and $2500 - wL > 625$

These conditions are met for a non-empty set of pairs (w, L). For example, if $L = 50$, then $w \in [25, 37.5]$, or if $L = 100$, then $w \in [12.5, 18.75]$.

Repeated games: Intuition

Consider a game such as the Prisoner's dilemma.

		Player 2	
		Talk	Not talk
Player 1	Talk	1, 1	7, 0
	Not talk	0, 7	4, 4

The "one shot game" implies (Talk, Talk). What if the game is repeated, say, 10 times?

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Reason with backward induction:

What happens in the 10th period? –> The last stage is played as a one shot game

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Then players may cooperate.

How cooperation is achieved? Through punishment threats: "*I cooperate as long as you cooperate, otherwise I will defect forever*".

If players care sufficiently for their future payoffs, then they might prefer a flow of payoffs such as 4, 4, 4, 4, 4, ..., to the payoff from cheating today (Talk), 7, 1, 1, 1, 1, 1, ...

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