Inter-temporal choice

Week IX

Marco Pelliccia

Readings: Perfloff Ch. 16, and Part IV of *Macroeconomics* of S.D.Williamson.

Outline

- Present Value and Future Value
- Inter-temporal Budget Constraint
- Inter-temporal Choice

Present Value and Future Value

- We usually value future consumption less than present one.

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- We define the **discount rate** the rate reflecting the relative value an individual places on future consumption compared to current consumption.

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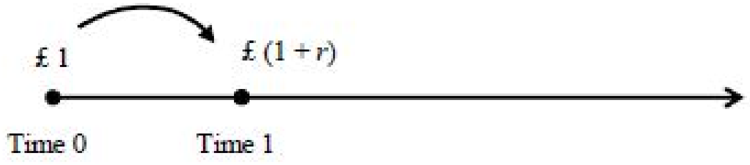
- How much is $1 worth in one year's time?
- **Future Value:**

$$\$1 \cdot (1 + r) = \$(1 + r)$$



Present Value and Future Value (cont.)

- Follow the same reasoning in the opposite direction: what is $\$1$ in one year's time worth today?

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- **Present Value:**

$$\$\frac{1}{1+r}$$

Present Value and Future Value (cont.)

- Why?!

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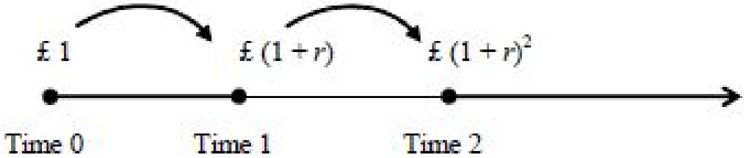
- If I invest $\$\frac{1}{1+r}$ today, in one year time it will be worth $\$\frac{1}{1+r}(1+r) = \$1$.

Present Value and Future Value (cont.)

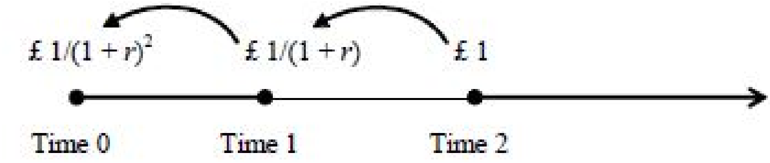
- Why?!Opportunity cost!
- If I invest $\$\frac{1}{1+r}$ today, in one year time it will be worth $\$\frac{1}{1+r}(1+r) = \$1$.
- If I invest $\$1$ today, in two years my investment is worth

$$1 \cdot (1+r) \cdot (1+r) = (1+r)^2$$



Present Value and Future Value (cont.)

- The present value of $1 to be cashed in two years is $\frac{1}{(1+r)^2}$



More in general we can say that

- The future value (FV) of $\$1$ in n years' time is

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- The future value of investment k in n years' time is

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Even more general,

$$FV = PV(1 + r)^n$$

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- A "pure discount bond" or zero coupon bond pays back its face value in m periods and nothing before then.
- We can simplify by assuming that all such bonds have face value of $\$1$ and that the market interest rate is constant and equal to r.
- What is the price today of a pure discount bond that pays back in m years?

Zero Coupon Bonds (cont.)

An m-period discount bond:

$$PV(m) = \frac{1}{(1+r)^m}$$

Coupon Bonds

Interest payments of c per year, and pays back its face value of $\$1$ plus a final coupon c in m years' time.

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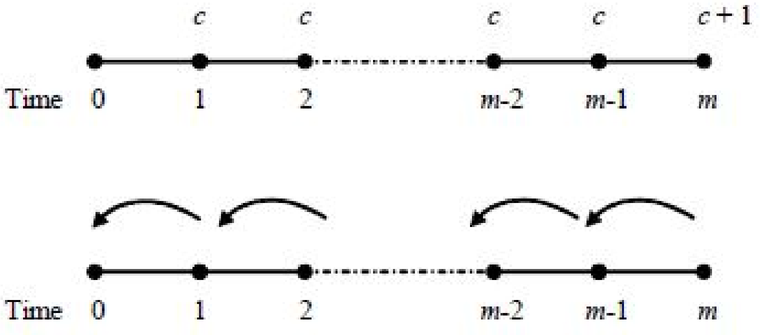

Coupon Bonds

Cash flow and discounted cash flow.

$$PV(c, m) = \frac{c}{(1+r)} + \frac{c}{(1+r)^2} + \cdots + \frac{c}{(1+r)^{m-1}} + \frac{c+1}{(1+r)^m}$$

Perpetuity

Interest payments of c per year, and never pays back its face value of $\$1$:

$$PV(c,\infty) = \frac{c}{(1+r)} + \frac{c}{(1+r)^2} + \cdots =$$

$$PV(c,\infty) = \sum_{t=1}^{\infty} \frac{c}{(1+r)^t} = c\sum_{t=1}^{\infty} \frac{1}{(1+r)^t}$$

Perpetuity (cont.)

We know that

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Therefore, we obtain

$$PV(c, \infty) = \frac{c}{r}$$

Valuing Investment Projects

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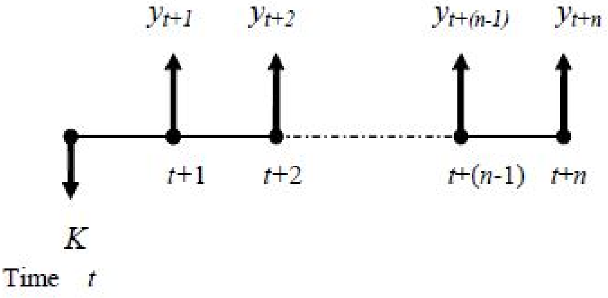
Assume that a project requires a capital K today (time t), and is expected to yield income flows in next n periods as follows:



Valuing Investment Projects (cont.)

Is the project worth pursuing?

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- We start computing the Present Value of the project:

$$PV = \frac{y_{t+1}}{1+r} + \frac{y_{t+2}}{(1+r)^2} + \frac{y_{t+3}}{(1+r)^3} + \cdots + + \frac{y_{t+n-1}}{(1+r)^{n-1}} + \frac{y_{t+n}}{(1+r)^n}$$

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- We then compare the PV of the project to the initial investment:
  - If $PV > K \Rightarrow$ we should invest
  - If $PV < K \Rightarrow$ we should not invest

Valuing Investment Projects (cont.)

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Note that none of the results so far depend on preferences!

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How do we introduce individual preferences over inter-temporal consumption?

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- Similar problem to the two-good model we have seen in the first part of the course.
- We relax the assumption which was reducing the problem to a single period one in order to analyse the impact of "time" and time preferences on the decision maker's choice.
- Still we assume no uncertainty.
- Two periods ("today and tomorrow"). The decision maker (DM) has income in each period equal to y_1 and y_2. He can consume c_1 and c_2 shares of his income in the two periods, or alternatively save s_1 and s_2.

Inter-temporal Budget Constraint (cont.)

- Therefore, it is clear for example that if $c_1 = y_1$, or the DM is consuming all his income of the first period, then $s_1 = 0$, or he will not able to save anything.

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- But if $s_1 = 0$, it means that the following period he can consume, at most, the income in period two.
- How can we construct the budget constraint (per period)?

Inter-temporal Budget Constraint (cont.)

- Per period budget constraint:

$$y_1 = c_1 + s_1 \tag{1}$$

Inter-temporal Budget Constraint (cont.)

- Per period budget constraint:

$$y_1 = c_1 + s_1 \tag{1}$$

$$y_2 + (1 + r)s_1 = c_2 + s_2 \tag{2}$$

Inter-temporal Budget Constraint (cont.)

If the DM lives only for two periods, we can say that $s_2 = 0$. Therefore, replacing (1) into (2), we obtain,

$$y_2 + (1+r)(y_1 - c_1) = c_2 + 0$$

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which we can write as

$$c_2 = y_2 + (1 + r)y_1 - (1 + r)c_1$$

Inter-temporal Budget Constraint (cont.)

We can interpret this as an inter-temporal budget constraint

$$c_2 = \underbrace{y_2 + (1+r)y_1}_{\text{Lifetime Icome}} - \underbrace{(1+r)}_{\text{Relative price of consumption today}} \cdot c_1$$

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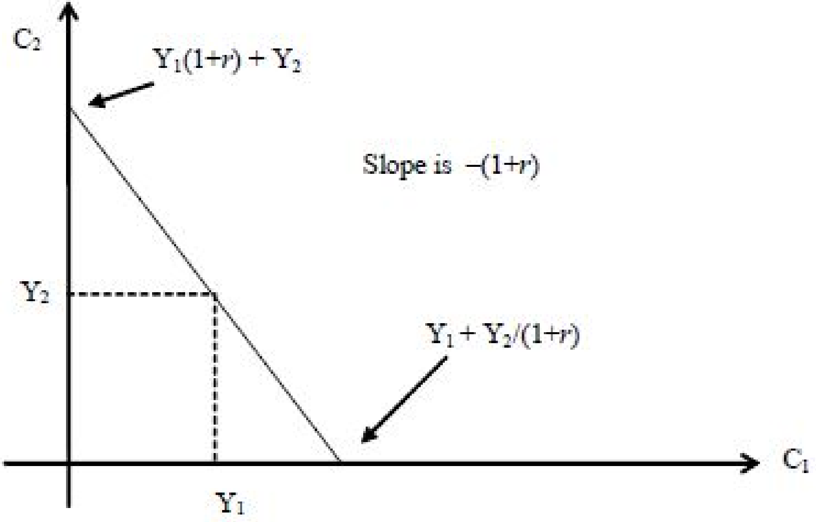
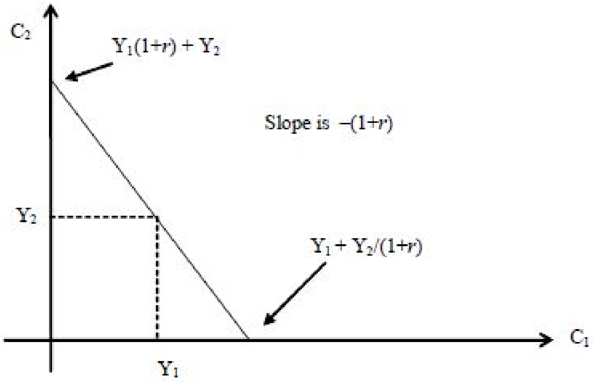
Recall that the 1-period budget constraint was $q_2 = \frac{y}{p_2} - \frac{p_1}{p_2} q_1$.

Inter-temporal Budget Constraint (cont.)

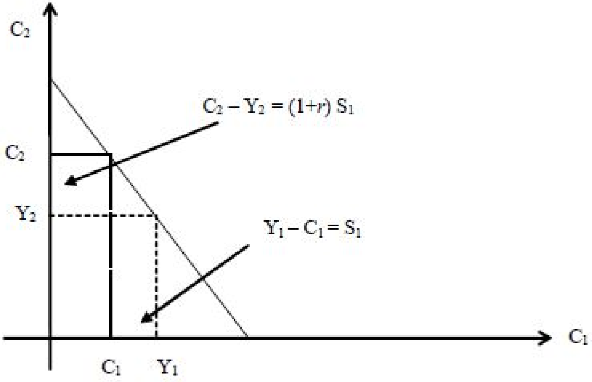


Inter-temporal Budget Constraint (cont.)



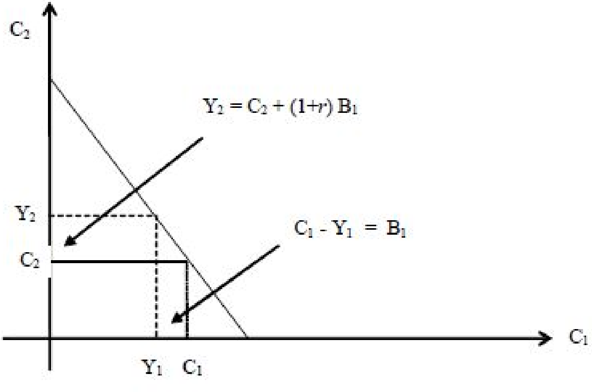
If no savings in period 1, $s_1 = 0$, then consumption is $c_1 = y_1$ and $c_2 = y_2$.

Inter-temporal Budget Constraint (cont.)



If DM saves in period 1, $s_1 > 0$, then $c_1 < y_1$ and $c_2 > y_2$.

Inter-temporal Budget Constraint (cont.)



If DM borrows at rate r against the future income, or $c_1 > Y$.

Inter-temporal Budget Constraint (cont.)

Note that we can rewrite the inter-temporal budget constraint in terms of present values:

$$\underbrace{C_1 + \frac{C_2}{1+r}}_{PV(C)} = \underbrace{Y_1 + \frac{Y_2}{1+r}}_{PV(Y)}$$

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Any combination of Y_1 and Y_2 with the same present value is on the same budget constraint.

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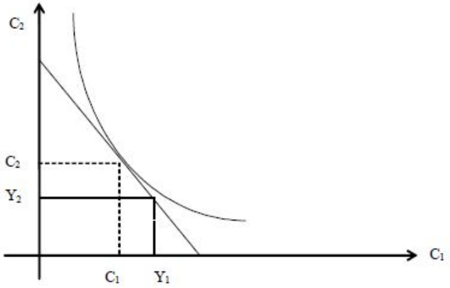

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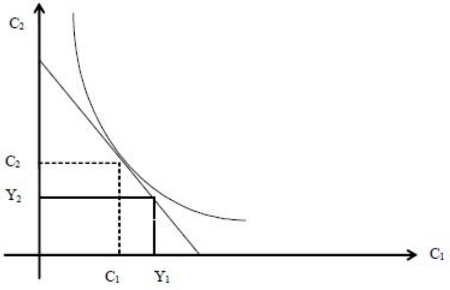

Optimal choice$\Rightarrow$ Tangency condition!

Inter-temporal Choice (cont.)

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- If different time profile of income but same PV, would optimal consumption change?
- Would saving change?

Impact of Rise of Income

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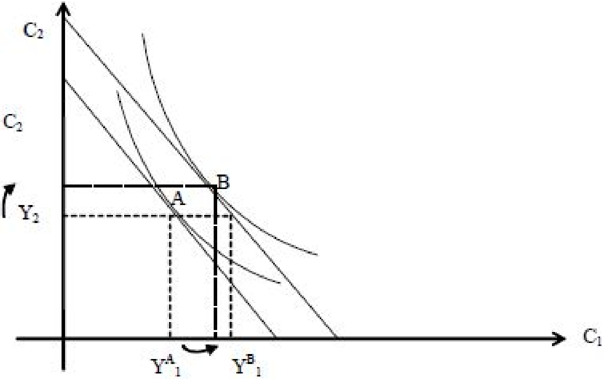
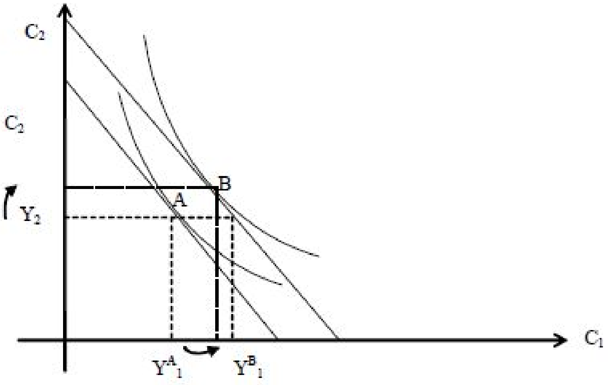
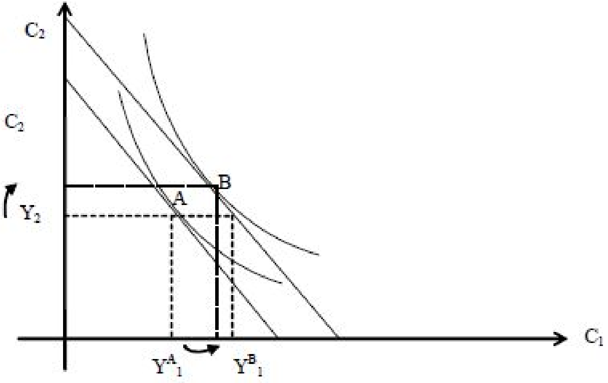


Impact of Rise of Income



Consumption increases in both periods: part of the income rise is saved for the future.

Impact of Rise of Income



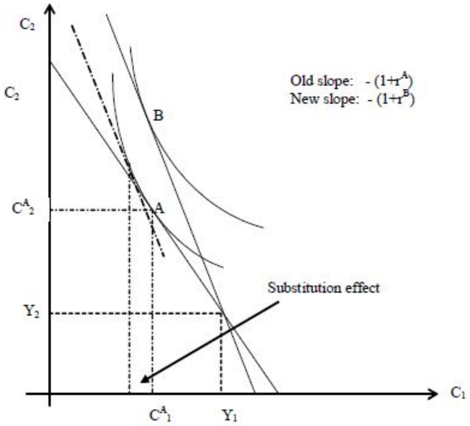
Consumption smoothing!

Impact of Rise of Income (cont.)

- What if y_1 is unchanged but y_2 rises?

Impact of Rise of r for saver

- Suppose that the DM was a saver ($s_1 > 0$). What happens if r increases?



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- By Substitution effect, a saver saves more (consume less)
- By Income effect, saver can now "afford" to save less.
- Final result will depend on preferences. In general a saver is made better off by a rise of r.

Impact of Rise of r for saver (cont.)

- Would a saver ever become a borrower?

Impact of Rise of r for saver (cont.)

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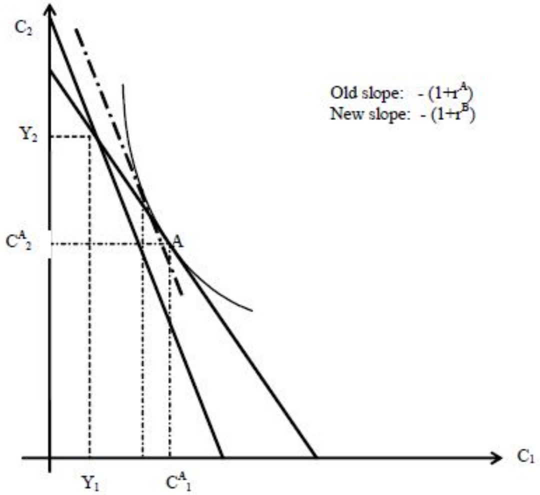
Impact of Rise of *r* for borrower



Impact of Rise of r for borrower (cont.)

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Impact of Rise of r for borrower (cont.)

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- By Substitution effect, a borrower will borrow less (and save more).
- Income effect goes in the same direction: borrower will borrow less.