Microeconomic Analysis

Seminar 1

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Basics of Preference Relations

Assume that our consumer chooses among L commodities and that the commodity space is $X \in \mathbb{R}^L_+$.

The basic idea of the preference approach is that given any two bundles, we can say whether the first is ***"at least as good as the second"***.

We denote this relation with $\succeq$. So if $x \succeq y$, that means that x is at least as good as y.

- We can also write $y \not\succeq x$ to say y is ***no better than*** x.
- If $x \succeq y$ **and** $y \succeq x$, we say that the consumer is ***indifferent*** between x and y, or symbolically $x \sim y$.
- The indifference curve I_y is the set of all bundles which are indifferent to y. That is, $I_y \equiv \{x \in X | x \sim y\}$.
- If $x \succeq y$ and **not** $y \succeq x$ ($y \not\succeq x$), we say that $x \succ y$, or x is ***strictly preferred*** to y.

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There are potentially various properties but we are particularly interested on **completeness** and **transitivity**.

- A binary relation $\succeq$ between two bundles $x, y \in X$ is **complete** if either $x \succeq y$, or $y \succeq x$.
- A binary relation $\succeq$ between three bundles $x, y, z \in X$ is **transitive** if $x \succeq y$ and $y \succeq z$ implies $x \succeq z$.

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...we can define a preference relation $\succeq$ ***rational*** if it is complete and transitive.

Monotonicity

Definition

A preference relation $\succeq$ is **monotone** if $x \succ y$ for any x and y such that $x_l > y_l$ for $l = 1, ..., L$. It is **strongly monotone** if $x_l \geq y_l$ for all $l = 1, ..., L$ and $x_l > y_l$ for some $j = 1, ..., L$ implies $x \succ y$.

Examples

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- The preferences represented by $x_1 + x_2$ satisfies *strong monotonicity*.

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It is clear that if the preference relation $\succeq$ is monotone or strictly monotone, the decision maker will choose a bundle on the boundary of the Walrasian budget set

Nonsatiation

Definition

A preference relation $\succeq$ satisfies **local nonsatiation** if for every x and any $\epsilon > 0$ there is a point y such that $||x - y|| \geq \epsilon$ and $y \succ x$.

Nonsatiation (cont.)

Two consequences,

- The DM will choose bundles in the boundary of the Walrasian budget set.
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- Local nonsatiation admits the possibility of satiation in one commodity
- The preference relation is globally nonsatiated if for all bundles $x \in X$ there exists $y \in X$ such that $y \succ x$.

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In Economics we often take the set X to be a infinite subset of a Euclidean space (technical assumption). The basic intuition is that for $a, b \in X$ if $a \succeq b$, then "small" deviations from a or from b will not reverse the ordering. Formally,

Definition

A preference relation $\succeq$ on X is **continuous** if whenever $a \succ b$, there are neighborhoods (balls) B_a and B_b around a and b, respectively, such that for all $x \in B_a$ and $y \in B_b$, $x \succ y$.

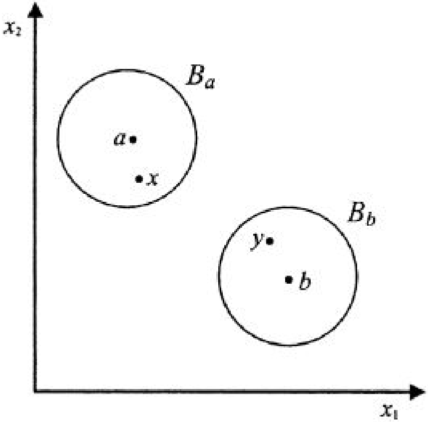

Figure: Continuity.

We can define the **upper level set** of x the set of all points that are at least as good as x, or $U_x \equiv \{y \in X | y \succeq x\}$. Similarly the **lower level set**, the set of all points that are no better than x, or $L_x \equiv \{y \in X | x \succeq y\}$.

Recall that a set is **convex** if for any two points $x, y \in X$, any point $z = \lambda x + (1 - \lambda)y$ with $\lambda \in [0, 1]$ is also in X.

Assuming convexity on the preference relation $\succeq$, and in particular strictly convexity (given any two distinct bundles $y \neq z$ such that $y \succeq x$ and $z \succeq x$, $\lambda y + (1 - \lambda)z \succ x$), implies "bowed upward" indifference set!

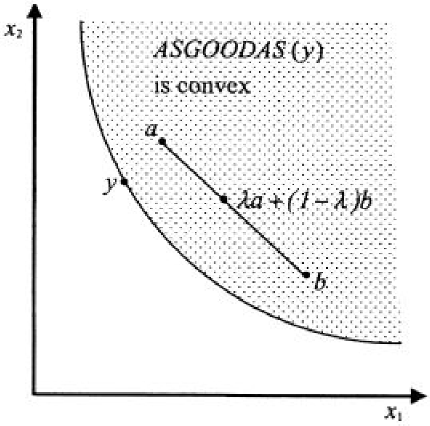

Figure: Convex set.

Utility function

The Utility function $U(x)$ assigns a number to every consumption bundles $x \in X$ and can **represent** preference relation $\succeq$ if for any $x, y \in X$, $U(x) \geq U(y) \iff x \succeq y$.

If you know the utility function that represents a consumer's preferences, you can analyze these preferences by deriving properties of the utility function!

Utility function (cont.)

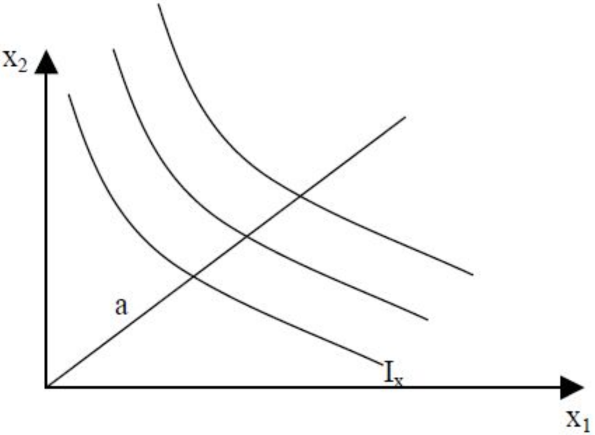

Figure: Ranking Indifference Curves

Utility function (cont.)

We can identify each I_x by its distance from the origin $(0, 0)$ along the line $x_2 = x_1$. The (unique) number associated with each I_x is the utility of x.

Utility as an ordinal concept

The number assigned to each I_x is essentially "arbitrary". Any assignment of numbers would do as long as the order of the numbers assigned to various bundles is not disturbed!

- If $U(x)$ represents $\succeq$ and $f(\cdot)$ is a monotonically increasing function, then $V(x) = f(U(x)$ also represents $\succeq$.

Example

Consider the utility function (Cobb-Douglas) $U(x_1, x_2) = x_1^a x_2^{(1-a)}$, and the monotonically increasing function $f(z) = \ln(z)$.

Then, the alternative utility function $V(x_1, x_2) = \ln(x_1^a x_2^{(1-a)}) = a\ln(x_1) + (1-a)\ln(x_2)$ represents the same preferences as $U(\cdot)$

Utility as an ordinal concept (cont.)

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- The difference between two utilities associated to two distinct bundles does not mean anything!

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Some basic property of $U(\cdot)$

- If preferences $\succeq$ are convex, then the indifference curves will be convex, as will be the upper level sets.
- When a function's upper level sets are always convex, we say that that function is **quasiconcave**.

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Example

Consider the Cobb-Douglas utility function $U(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$.

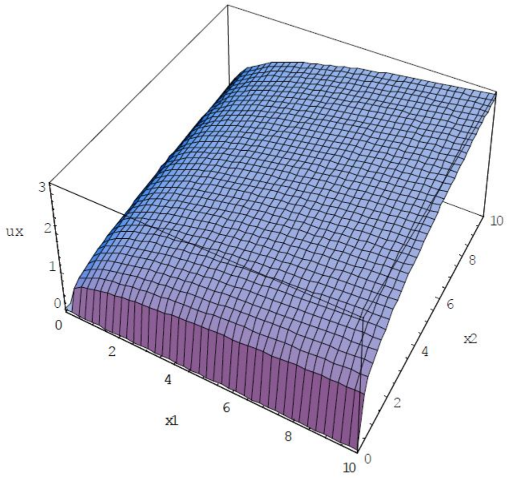

Figure: $U(\cdot)$ function.

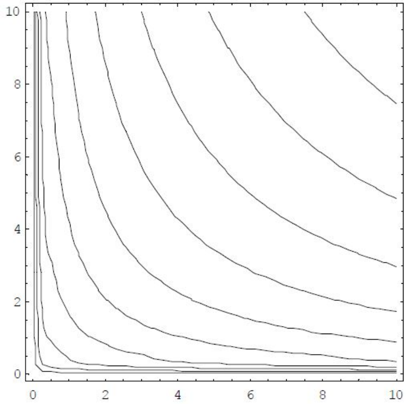

Figure: Level sets of $U(\cdot)$.

- The indifference curves are **convex.**
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Quasiconcavity is a weaker condition than concavity.

- Concavity is a cardinal concept while quasiconcavity is an ordinal one (it talks about the shape of the indifference curves and not about the number assigned to them).

Example

Consider the Cobb-Douglas utility function $V(x_1, x_2) = x_1^{\frac{3}{2}} x_2^{\frac{3}{2}}$.

- $V(\cdot)$ is not concave while $U(\cdot)$ was a concave function.

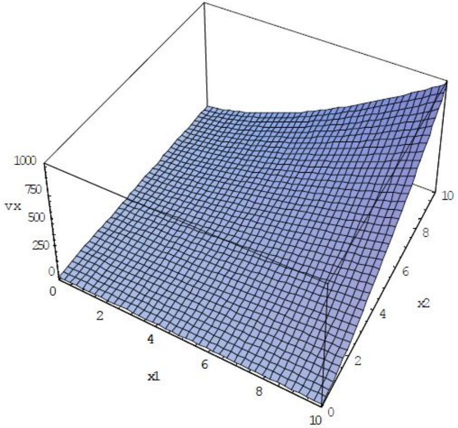

Figure: $V(\cdot)$ function.

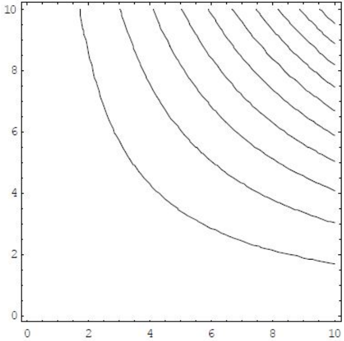

Figure: Level sets of $V(\cdot)$.

Even if the $V(\cdot)$ is not concave, the level sets are still convex! Hence, $V(\cdot)$ is quasiconcave.

Quasiconcavity is about the shape of level sets, not about the curvature of the 3D graph of the function.

Some fun...Relativity

This was a real *Economist's* offer (taken from Dan Ariely's book):

Figure: *Economist* subscription offer

Some fun...Relativity

Summarizing the choice options:

1. Internet only $59
2. Print only $125
3. Print + Internet $125

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Most people do not know what they want unless they see it in context!