Microeconomic Analysis

Seminar 6

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2 questions...

Consider an industry with $N = 3$ firms, each having marginal costs equal to 0. The inverse demand curve is

$$P(Q) = 60 - Q$$

with $Q = q_1 + q_2 + q_3$ is total output.

If each firm behaves as a Cournot competitor, what is the firm 1's best response function?

The profit of firm 1 is given by
$\pi_1 = P(Q)q_1 = (60 - q_1 - q_2 - q_3)q_1$. Therefore the FOC will lead

$$60 - 2q_1 - q_2 - q_3 = 0 \Longrightarrow q_1 = 30 - \frac{1}{2}(q_2 + q_3)$$

Calculate the Cournot equilibrium.

The firms are identical, therefore we need to solve the following system

$$\begin{cases} q_1 = 30 - \frac{1}{2}(q_2 + q_3) \\ q_2 = 30 - \frac{1}{2}(q_1 + q_3) \\ q_3 = 30 - \frac{1}{2}(q_2 + q_1) \end{cases}$$

which leads to $q_1^* = q_2^* = q_3^* = 15$, price $P = 60 - 45 = 15$, and profit $\pi_i = 15 \times 15 = 225$ for $i \in \{1, 2, 3\}$.

Firm 2 and 3 decide to merge (marginal cost is still 0). Calculate the new industry equilibrium. Is firm 1 worse or better? Firms 2 and 3?

If firms 2 and 3 merge, the best reply function of 1 becomes $q_1 = 30 - (1/2)\tilde{q}$ with $\tilde{q}$ the quantity produced by 2 and 3, and $\tilde{q} = 30 - (1/2)q_1$.

Solving the system we obtain $q_1^* = 20$ and $\tilde{q} = 20$, where we can assume $q_2 = q_3 = \tilde{q}/2$. Therefore, the price in the market will be $P = 60 - 40 = 20$, and the profits $\pi_1 = 20 \times 20 = 400$ and $\pi_2 = \pi_3 = 20 \times 20/2 = 200$.

The profit of firm 1 increases while it is lower for 2 and 3.

Would it be good to form a cartel composed by all three firms?

The profit of the new monopolist will be $(60 - Q_m)Q_m$, and therefore the FOC

$$60 - 2Q_m = 0 \Longrightarrow Q_m^* = 30$$

The price then will be $P = 60 - 30 = 30$ and the profit for each firm composing the cartel, $\pi_i = 30 \times 30/3 = 300$. Therefore, it is convenient to form a cartel!

Suppose that firm 1 can commit in advance to a certain level of output. What is the optimal level of q_1? Calculate the profit of firm 1.

Firms 2 and 3 observe the quantity q_1 and compete between them *à la* Cournot. The best reply function for 2 is

$$q_2 = 30 - \frac{1}{2}q_1 - \frac{1}{2}q_3$$

Define $30 - \frac{1}{2}q_1 \equiv A$. Then we have $q_2 = A - \frac{1}{2}q_3$ and $q_3 = A - \frac{1}{2}q_2$. Solving the system, we obtain $q_2^* = q_3^* = (2/3)A$ and then $Q_{2,3} = (4/3)A$. In particular,

$$Q_{2,3} = \frac{4}{3}(30 - \frac{1}{2}q_1) = 40 - \frac{2}{3}q_1$$

The demand faced by firm 1 is

$$P = 60 - q_1 - 40 + \frac{2}{3}q_1$$

and profit function $\pi_1 = (60 - 40 - \frac{1}{3}q_1)q_1$. Solving by FOC we obtain $q_1^* = 30$.

Firms 2 and 3 will produce then $q_2^* = q_3^* = 10$. Thus, $P = 60 - 50 = 10$, $\pi_1 = 300$ and $\pi_{2,3} = 100$. Comparing the profits with the classic Cournot equilibrium (question b), we note that firm 1 benefits from the "leader" position, while firms 2 and 3 are worse.

Suppose that a new technology is created that allows for owners of a video game to copy and resell digital material for free. Suppose that a bootlegger has some probability of getting caught $\alpha(x)$ in which case he will have to pay a fine $F > 0$ and give up the revenues received from the copy. The probability $\alpha(x)$ is an increasing function of x, the number of copies, i.e. the more copies you make, the more likely you are to get caught. We assume $\alpha(x)$ differentiable. The profit of a bootlegger is

$$\max_x [1 - \alpha(x)]px - \alpha(x)F$$

Assume "free-entry" in the market of bootleggers.

1. Show that the scale of operation, x^*, is independent of the size of the fine F.
2. Compute the price charged by each bootlegger.
3. Suppose that the developer of the video-game faces a fixed cost K but zero variable costs $c(q) = 0$. We know that in the long-run, the developer produces the video-game if and only if $pD(p) \geq K$. Show that

$$\alpha F \geq (1-\alpha)\frac{x^*}{D(p^*)}K$$

4. Based on your answer for c), would you expect the fine in a country with low α to be higher or lower than the one charged in a country where α is relatively high?

1. **Show that the scale of operation, x^*, is independent of the size of the fine F.**

Solving the FOC yields

$$\begin{aligned} p - p\alpha - px\alpha' - F\alpha' &= 0 \\ p(1-\alpha) - \alpha'(F + px) &= 0 \end{aligned}$$

We need to explicate x as uniquely function of $\alpha(x)$. We need then another condition $\rightarrow$ We can use the entry condition for each bootlegger!

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$$\text{Profit} = 0 \Longrightarrow [1 - \alpha(x)]px - \alpha(x)F = 0 \tag{1}$$

In particular, we can see from (1) that $px + F = \dfrac{px}{\alpha(x)}$. We can replace it into the FOC.

$$p(1-\alpha) - \alpha' \frac{px}{\alpha} = 0$$

so we can find $x^* = \dfrac{(1-\alpha(x))\alpha(x)}{\alpha'(x)}$. In other words, the quantity copied by each bootlegger is not conditioned by the fine F but only by the probability to be caught $\alpha(x)$.

1. **Compute the price charged by each bootlegger.**

Plugging x^* into the entry restriction we can find p^*,

$$[1-\alpha(x)]px^* - \alpha(x)F = 0 \Longrightarrow p^* = \frac{\alpha(x^*)F}{(1-\alpha(x^*))x^*}$$

1. **Suppose that the developer of the video-game faces a fixed cost K but zero variable costs $c(q) = 0$. We know that in the long-run, the developer produces the video-game if and only if $pD(p) \geq K$. Show that**

$$\alpha F \geq (1-\alpha)\frac{x^*}{D(p^*)}K$$

$$p^* D(p^*) \geq K \Longrightarrow \frac{\alpha(x^*)F}{(1-\alpha(x^*))x^*}D(p^*) \geq K$$

Rearranging we obtain,

$$\alpha(x^*)F \geq \frac{(1-\alpha(x^*))x^* K}{D(p^*)}$$

1. **Based on your answer for c), would you expect the fine in a country with low α to be higher or lower than the one charged in a country where α is relatively high?**

From the previous condition (in part c)), we see that if α is particularly small, the conditions guaranteeing the existence of a market for the developers would require relatively high fine F.