An Introduction on Comparison of Payoff Distributions

Seminar IX

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There are two natural ways to compare them:

- According to the level of returns
- According to the level of *riskiness* or *dispersions* of returns

Thus we want to see when

- A cumulative distribution $F(\cdot)$ yields unambiguously higher returns than $G(\cdot)$.
- $F(\cdot)$ is unambiguously less risky than $G(\cdot)$.

PDF and CDF

When there is a continuum of outcomes, we will represent a lottery as a distribution over the outcomes. The probability density function (pdf) is defined such that

$$Pr(a \leq x \leq b) = \int_a^b f(x)dx$$

PDF and CDF

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$$Pr(a \leq x \leq b) = \int_a^b f(x)dx$$

Therefore, the expected utility of a distribution $f(\cdot)$ is given by

$$U(f) = \int_{-\infty}^{+\infty} u(x)f(x)dx$$

PDF and CDF

It could be also useful to write a lottery in terms of its cumulative distribution function (cdf) rather than its pdf. The cdf of a random variable is given by

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When we write the cdf to represent the lottery, we will write the expected utility of F as

$$\int_{-\infty}^{+\infty} u(x)dF(x)$$

PDF and CDF

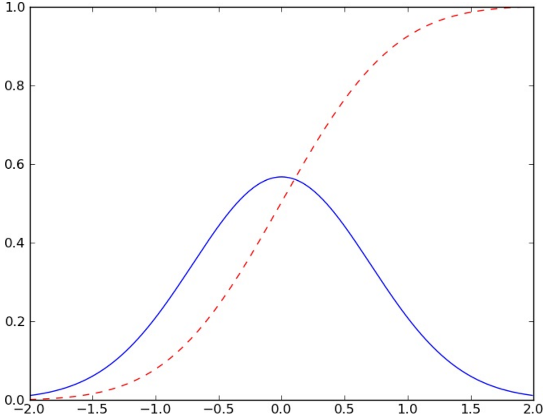

Figure: Pdf and Cdf of a Standard Normal distribution

Risk propensity

We can rewrite the conditions for the risk-propensity of a decision maker as

Risk-averse:

$$\int u(x)dF(x) \leq u\left(\int xdF(x)\right)$$

Risk-lover:

$$\int u(x)dF(x) \geq u\left(\int xdF(x)\right)$$

Risk-neutral:

$$\int u(x)dF(x) = u\left(\int xdF(x)\right)$$

Stochastic Dominance-FOSD

Definition

The distribution $F(\cdot)$ first-order stochastically dominates $G(\cdot)$ if, for every nondecreasing function $u : \mathbb{R} \longrightarrow \mathbb{R}$, we have

$$\int u(x)dF(x) \geq \int u(x)dG(x)$$

In particular, $F(\cdot)$ FOSD $G(\cdot)$ if and only if $F(x) \leq G(x) \ \ \forall x$.

Stochastic Dominance-FOSD

In other words, $F(\cdot)$ FOSD $G(\cdot)$ if for all x the graph of $F(\cdot)$ is below the one of $G(\cdot)$.

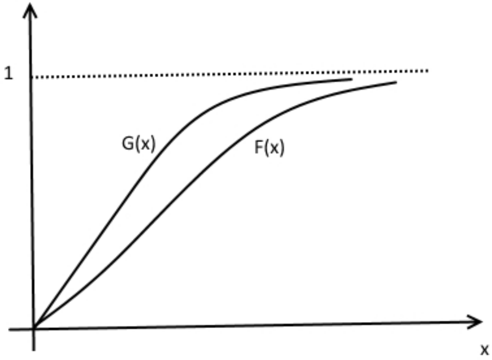

Figure: $F(\cdot)$ FOSD $G(\cdot)$

Stochastic Dominance-SOSD

Suppose we want to compare two distributions according to their relative dispersion.

Stochastic Dominance-SOSD

Suppose we want to compare two distributions according to their relative dispersion.

To simplify the analysis, assume that $F(\cdot)$ and $G(\cdot)$ have the same mean, that is $\int x dF(x) = \int x dG(x)$.

Definition

The distribution $F(\cdot)$ for a risk-averse DM second-order stochastically dominates $G(\cdot)$ if, for every nondecreasing function $u : \mathbb{R}_+ \longrightarrow \mathbb{R}$, we have

$$\int u(x) dF(x) \geq \int u(x) dG(x)$$

In particular, $G(\cdot)$ is a mean-preserving spread of $F(\cdot)$.

Stochastic Dominance-SOSD

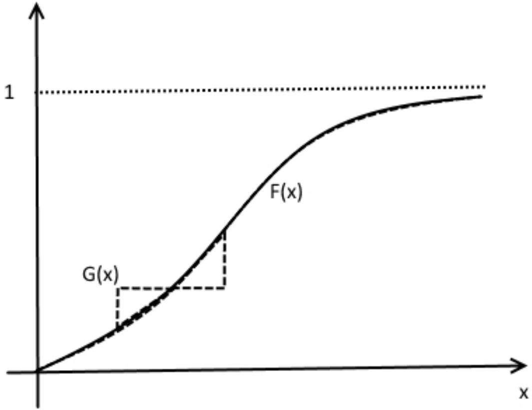

Figure: $F(\cdot)$ SOSD $G(\cdot)$

Recap.

- *First-order stochastic dominance:* when a lottery F dominates G in the sense of first-order stochastic dominance, the decision maker prefers F to G regardless of what u is (his risk propensity), as long as it is weakly increasing.
- *Second-order stochastic dominance:* when a lottery F dominates G in the sense of second-order stochastic dominance, the decision maker prefers F to G as long as he is risk averse and u is weakly increasing.