Public Goods in Networks

Yann Bramoullé
Université Laval

Rachel Kranton
University of Maryland

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Introduction

- We study the first model of strategic experimentation in social networks.

- Individuals experiment to obtain new information *and* benefit from their neighbors' experimentation.

- Learning from others is a main source of learning in many situations:
  - Consumer choice.
  - R&D spillovers between firms.
  - Innovation adoption.

- Extensive literature in sociology and in marketing. Recent studies in economics.

- For instance, consider the adoption of a new crop in rural areas of developing countries.

- Conley & Udry (2003)

  - Look at the adoption of pineapple for export in Ghana.

  - Collected precise data on communication links. E.g. “Have you ever gone to X for advice about your farm?”.

  - “Our findings suggest that a farmer increases his fertilizer use after someone *with whom he shares information* achieves higher than expected profits when using more fertilizer than he did.”

  - Evidence of social learning. Does not take place at the village level. Example of information network.

- Foster & Rosenzweig (1995)

  - Look at the adoption of high-yield rice and wheat in India in the 1970’s.

  - “We find that farmers with experienced neighbors are significantly more profitable than those with inexperienced neighbors.”

- “farmers tend to free-ride on the learning of others” and “curtail their own costly experimentation” following an increase in the rate of adoption of their neighbors.

- Evidence of social learning. Yields strategic experimentation.

- We combine the two sets of findings.

- We study how the shape of the communication network affects experimentation patterns and welfare.

- We find strong network effects:

  - On the overall level of experimentation. May be lower in denser networks.

  - On individual experimentation. May be lower for individuals with a more central position in the network.

  - On the experimentation pattern. Networks lead to specialization and effort inequality.

  - On welfare. Inequal efforts may yield higher welfare when individuals who experiment are well-connected.

- New links increase access to information, but decrease incentives to experiment and may lower welfare.

- Contribution of the paper:

  - Introduces network aspects to the literature on strategic experimentation, Bolton & Harris (1999).

  - Endogenizes the generation of information in models of learning in networks, Bala & Goyal (1998).

  - Advances the economic theory of networks.

  - Studies the first model where a good is non-excludable among linked individuals.

  - Develops a new research strategy: Builds families of graphs to model different social structures.

Model

- Simple model.
- n individuals, Set of agents $N = \{1,...,n\}$.
- Social network **g**, where $g_{ij} = 1$ indicates i and j are social neighbors.
- $N_i = \{j \in N - i : g_{ij} = 1\}$ Set of agents that are directly linked to agent i and $k_i = |N_i|$ number of i's neighbors
- Individuals can experiment to acquire information.
- Experimentation profile $\mathbf{e} = (e_1, ..., e_n)$. E.g. amount of land planted with a new crop.

- Individuals benefits from the experimentation results of their neighbors.

$$b\left(e_i + \sum_{j \in N_i} e_j\right)$$

where *b(.)* is increasing and concave.

- i.e., information diffuses one step, no decay.

- Constant marginal cost of experimentation *c*.

- Given **g**, individuals simultaneously choose their experimentation level e_i.

- Payoff for individual *i*:

$$U_i(\mathbf{e};\,\mathbf{g}) = b\left(e_i + \sum_{j\in N_i} e_j\right) - c_i e_i$$

Observe that:

$$\frac{\partial^2 U_i}{\partial e_i^{\,2}} = \frac{\partial^2 U_i}{\partial e_i \partial e_j} = b''\left(e_i + \sum_{j\in N_i} e_j\right) < 0$$

Thus e_i and e_j (local) strategic substitutes when $g_{ij} = 1$

- This defines a static game parametrized by the social network.
- How do the equilibria depend on the network?

Nash Equilibria

- Let e^* be such that $b'(e^*) = c$.

i.e.

$$e^* = b'^{-1}(c)$$

Experimentation for an isolated individual.

- Let $\bar{e}_i = \sum_{j \in N_i} e_j$ denote the information individual i receives from her neighbors, i.e. the total effort of i's neighbors.

Proposition: A profile ***e*** is a Nash equilibrium if and only if for every agent *i* either:

(1) $\bar{e}_i \geq e^*$ and $e_i = 0$

or

(2) $\bar{e}_i \leq e^*$ and $e_i = e^* - \bar{e}_i$

Proof. FOC: $$\frac{\partial U_i}{\partial e_i} = b'\left(e_i + \sum_{j \in N_i} e_j\right) - c = 0$$

This is equivalent to:

$$e_i + \sum_{j \in N_i} e_j = b'^{-1}(c) = e^*$$

$$\Leftrightarrow e_i = e^* - \sum_{j \in N_i} e_j$$

Therefore:

$$e_i = \begin{cases} 0 \quad if \quad e^* \leq \sum_{j \in N_i} e_j \\ \\ e^* - \sum_{j \in N_i} e_j > 0 \quad otherwise \end{cases}$$

- Abstract characterization is easy. Geometric characterization is difficult.

- Experimentation levels are strategic substitutes.

- Distinguish three types of equilibria:

  A profile **e** is *specialized* when:
  Individuals exert $e_i = 0$ or $e_i = e^*$
  The agent $e_i = e^*$ is a *specialist*.

  A profile **e** is *distributed* when all individuals experiment, i.e. every agent exerts some positive effort, $0 < e_i < e^*, \forall i \in N$

  Hybrid equilibria fall between these two extremes

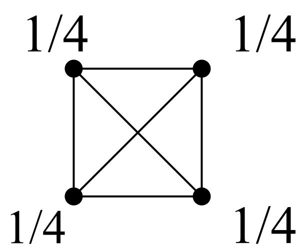
- Benefits for individuals who do not experiment may be greater than *b(e*)*.

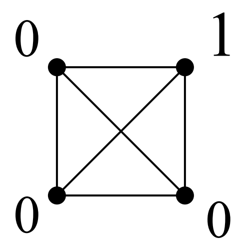
- Indicates potential gains from specialization.

Illustration on Simple Graphs

$e^* = 1$

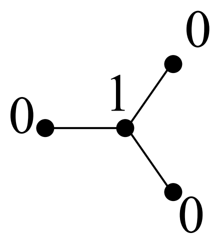
- Complete graph (Nash equilibria)

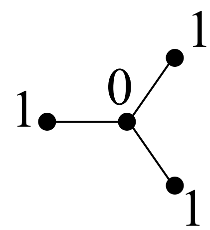




  - Symmetric, densely connected society.
  - Information is public.
  - Overall equilibrium experimentation (aggregate effort) is e^*, distributed in any way.

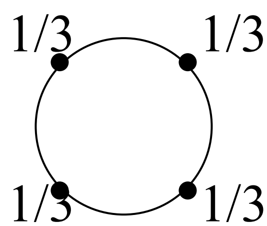
- Star

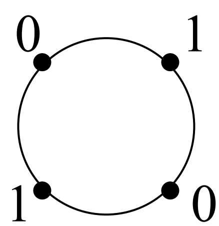




  - Asymmetric network.
  - All equilibria are specialized: (1) the center experiments, or (2) all agents in the periphery experiment.
  - In the right equilibrium, the center earns *b(3e*)*.

- Circle





  - Symmetric, not densely connected society.
  - Both distributed and specialized equilibria.
  - In the right equilibrium, individuals who do not search earn *b(2e*)*.

- What do we learn from these simple graphs?
  - The network is a main determinant of the equilibria.
  - The overall level of experimentation is usually indeterminate on incomplete networks.
  - Denser networks can lead to less overall experimentation.
  - Effort sharing is not always possible.
- In general networks, existence of equilibria guaranteed by standard arguments.
- However, this says nothing on their shape.

Definition: An independent set I of a network **g** is a set of players such that no two players who belong to I are linked, that is $\forall i, j \in I$ such that $i \neq j$, $g_{ij} = 0$. An independent set is maximal when it is not a proper subset of any other independent set.

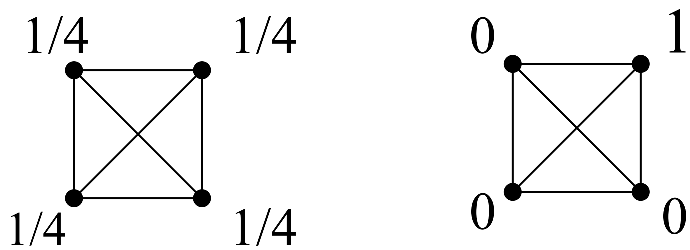
Any maximal independent set has the property that every player either belongs to it or is connected to a player that belongs to it.

For any player i, there exists a maximal independent set I of the network **g** such that i belongs to I. This implies that any network **g** possesses at least one maximal independent set.

Given a graph **g**, we can define **a maximal independent set of order *r*** such that any individual not in I is connected to at least r individuals in I. That is, for a maximal independent set of order r, agents outside the set can have more than r, but no less than r, connections to agents in the set.

The case $r = 1$ simply corresponds to maximal independent sets. While every graph contains maximal independent sets, not all graphs contain maximal independent sets of higher order.

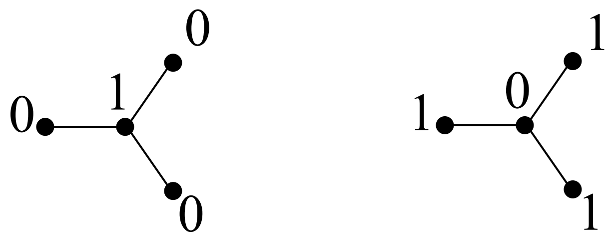
Consider first this Figure: complete graph with n=4



An independent set can include at most one player. There are thus four maximal independent sets, each including one player.

There is *no* maximal independent set of order r =2 or higher. This is a general property of complete graphs.

Consider now this Figure: a star network with n=3.

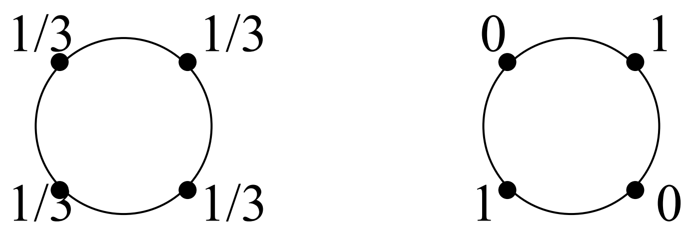


There are two maximal independent sets: the one that only includes the central player and the one that includes *both* peripheral players.
Observe that each peripheral player constitutes an independent set but it is *not* maximal.

There is however only one maximal independent set of order $r = 3$ composed by all peripheral players.

This is a general result of star-shaped graphs. If there are n players, then there is only one maximal independent set of order $r = n - 1$ composed of all peripheral players together.

Consider this Figure: a circle with n=4.



There are two maximal independent sets, each containing each player on opposite sides of the circle.

These two maximal independent sets are of order $r = 2$.

Go back to the model.

Because effort are strategic substitutes, maximal independent sets are a natural notion in this model. Indeed, in equilibrium, no two specialists can be linked. Hence, specialized equilibria are characterized by this structural property of a graph.

Specialists = *maximal independent set* of the graph.

Theorem 1: A specialized profile is a Nash equilibrium if and only if its set of specialists is a maximal independent set of the structure **g**. Since for every **g** there exists a maximal independent set, there always exists a specialized Nash equilibrium.

Proof:

Consider a specialized equilibrium where I is the set of specialists. Specialists play a best response if all their neighbors exert zero effort. This means that I is an independent set of the graph. A non specialist i plays a best response if

$$\sum_{j \in N_i} e_j \geq e^*$$

$$\Leftrightarrow \left| N_i \bigcap I \right| \geq 1$$

$$\Leftrightarrow \left| N_i \bigcap I \right| \geq r$$

This means that all players not in I are connected to at least r players in I. Combining both properties yields the result. Q.E.D.

Heuristic proof

- Take one agent *i*. Let her play $e*$. Let all her neighbors play *0*. Remove *i* and her neighbors.

- From remaining agents, take another agent *j*. Repeat the operation with *j*.

- Continue until all agents are covered.

- In the end, (1) no two specialists are linked, and (2) every non-specialist is connected to a specialist.

Q.E.D

- Specialized equilibria characterized by simple structural property of the graph.

Equilibrium selection: stable Nash equilibria

- Consider a simple notion of stability based on Nash tâtonnement.

See Fudenberg and Tirole (1991), *Game Theory*.

Definition: Define $f_i(\mathbf{e})$ as the best response of individual i to a profile $\mathbf{e} = (e_1,...,e_n)$ and define $\mathbf{f}$ as the collection of these individual best responses $\mathbf{f} = (f_1(\mathbf{e}),...,f_n(\mathbf{e}))$. Then, an equilibrium $\mathbf{e} = (e_1,...,e_n)$ is *stable* if and only if there exists a positive number $\rho > 0$ such that, for any vector $\boldsymbol{\varepsilon} = (\varepsilon_1,...,\varepsilon_n)$ satisfying $\forall i, |\varepsilon_i| \leq \rho$ and $e_i + \varepsilon_i \geq 0$, the sequence $\mathbf{e}^{(n)}$, defined by $\mathbf{e}^{(0)} = \mathbf{e} + \boldsymbol{\varepsilon} = (e_1 + \varepsilon_1,...,e_n + \varepsilon_n)$ and $\mathbf{e}^{(n+1)} = \mathbf{f}(\mathbf{e}^{(n)})$, converges to $\mathbf{e} = (e_1,...,e_n)$.

This (standard) notion leads to a strong result:

- Only specialized equilibria are stable.

This result rests on the *strategic substitutability* of efforts of linked players.

Consider an equilibrium where everyone exerts some effort, and decrease the effort of an individual *i* by a small amount. Her neighbor(s) will adjust by increasing their own efforts. This increase can lead *i* to reduce his effort even more. In this case, the initial equilibrium is not stable.

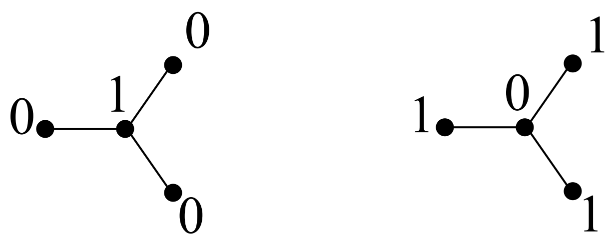
This process does not work in specialized equilibria when every agent *j* who exerts no effort is linked to two specialists. If we reduce the effort of these specialists, agent *j* will not adjust. He has access to two sources of information, and a small reduction will not lead him to increase his own effort.

Stable profiles thus correspond to maximal independent sets of order 2. Given a graph **g**, we
show a stable equilibria exists if and only if there is a maximal independent set of order 2.

Theorem 2: For any social structure **g**, an equilibrium is stable if and only if it is specialized and every non-specialist is connected to (at least) two specialists. Hence, there exists a stable equilibrium in a graph **g** if and only if it has a maximal independent set of order 2.

Thus an equilibrium is stable iff it is specialized and non-specialists are linked with at least two specialists.

Consider now the star network with n=3.



In both graphs, there is only one maximal independent set of order $r = 2$ composed by all peripheral players.

Consider the Nash equilibrium where the center exerts $e^{*} = 1$ and peripheral agents exert no effort (graph on the left).
The set of specialists is not a maximal independent set of order 2. Thus this equilibrium is *not* stable.

In contrast, consider the equilibrium where all peripheral agents exert effort (graph on the right). This equilibrium is stable, as the set of specialists is a maximal independent set of order 2.

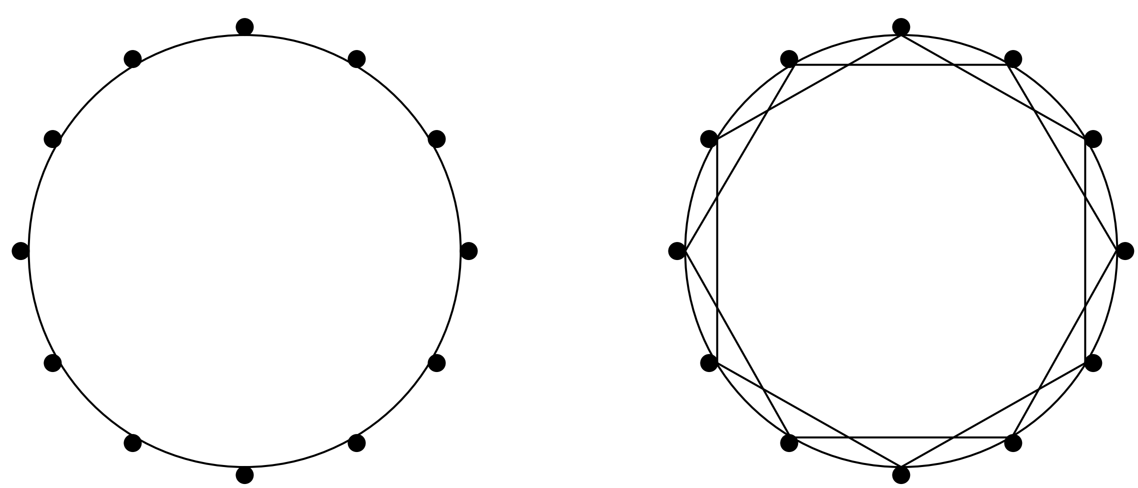
- E.g., on the star, experimentation by the center is not stable.
  → Better connected agents do less.

- Inequality in experimentation is a natural outcome of the network structure.

Models of Social Networks

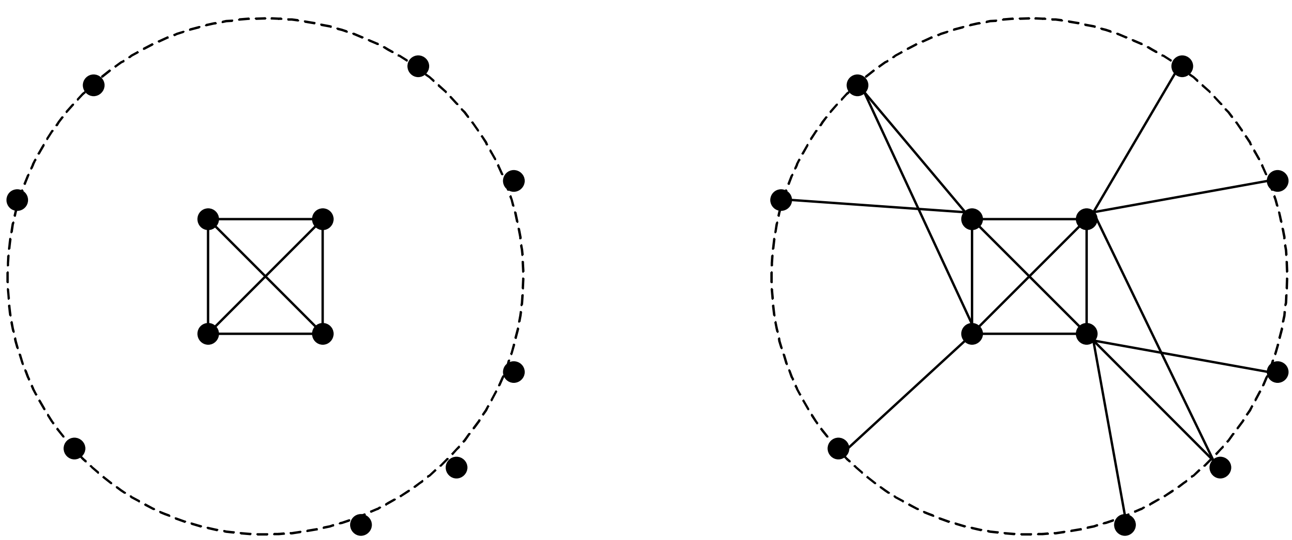
- In the paper, we build families of graphs representing different social structures.

- *Overlapping Neighborhoods*: symmetric structure where agents learn from those close in geographic or social space. Explore increasing levels of network density.



- *Communities/Bridges*: asymmetric structure with agents divided into disjoint communities. Explore increasing numbers of links – bridges – between communities.

- *Core-Periphery*: asymmetric, hierarchical structure where agents in periphery rely on core. Explore increasing density of links between core and periphery.



Welfare Analysis

- What are the welfare properties of the different equilibrium profiles?

- We adopt a simple utilitarian approach

$$W(\mathbf{e},\mathbf{g})=\Sigma_i\, U_i(\mathbf{e},\mathbf{g})$$

- Because of information externalities, no equilibrium yields first-best level of welfare.

- We study which Nash equilibria yield highest welfare.

  → *Second-best* profiles.

$$\frac{\partial W(\mathbf{e};\mathbf{g})}{\partial e_i} = b'\left(e_i + \sum_{j\in N_i} e_j\right) + \sum_{j\in N_i} b'\left(e_i + \sum_{j\in N_i} e_j\right) - c = 0$$

- Second term is *information premium* from specialization.

Compare with Nash equilibrium

$$\frac{\partial U_i}{\partial e_i} = b'\left(e_i + \sum_{j\in N_i} e_j\right) - c = 0$$

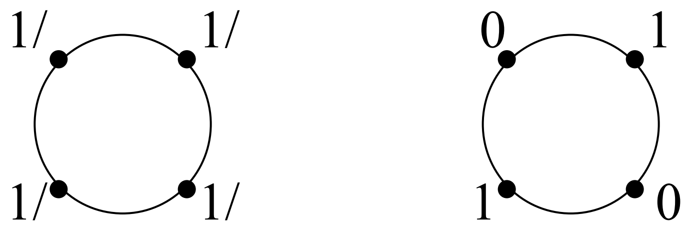
- A trade-off emerges :

* distributed equilibria have lower search costs.

* specialized equilibria can have information premia.

Overall experimentation is greater in specialized equilibria.

E.g. Circle



- Welfare of distributed equilibria

  $$4\ b\ (e^*) - (4/3)\ ce^*$$

- Welfare of specialized equilibria

  $$4\ b\ (e^*) + 2\ [b(2e^*) - b(e^*)] - 2\ ce^*$$

- Specialized equilibria are second-best when information premium exceeds additional search costs

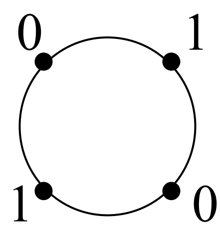
  $$[b(2e^*) - b(e^*)] > (1/3)\ ce^*$$

Proposition: Specialized equilibria yield greater welfare if: (1) more information is sufficiently valuable, and (2) specialists are sufficiently well-connected.

$$\sum_{i\ specialist} k_i > n - 1$$

where k_i is the number of neighbors of i.

E.g. In circle, specialists together have 4 neighbors, and $n - 1 = 3$



- Similar method to compare specialized equilibria - count number of links between specialists and non-specialists.

What is the effect of adding a link?

- Consider a graph **G** and agents *i* and *j* not linked in **G**.

- We say a link between *i* and *j* leads to a welfare loss when second-best welfare level for **G** +*ij* is lower than that for **G**.

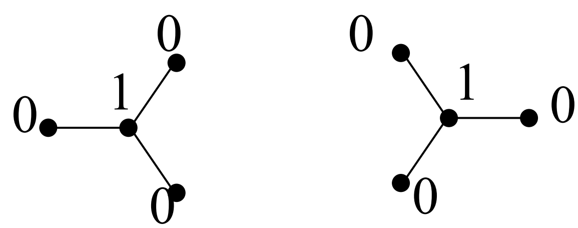
- Consider a second-best profile for **G:**

- *Benefit of New Link*: If *i* or *j* does not experiment, equilibrium is preserved. Link adds new source of information.

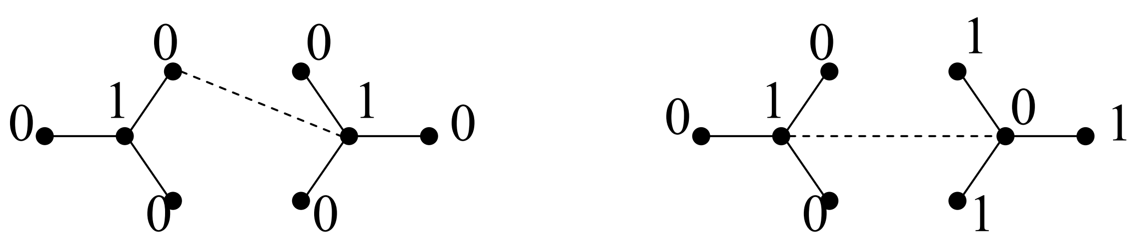
- *Cost of New Link*: If both *i* and *j* experiment, equilibrium is not preserved. Link is new source of information but leads to a loss of experimentation.

Proposition: A necessary condition for a loss in welfare is both agents experiment in all second-best profiles for **G**.

- Illustration: Two Stars



- Experimentation by center individuals is unique second-best profile.



- Linking center to periphery increases welfare.

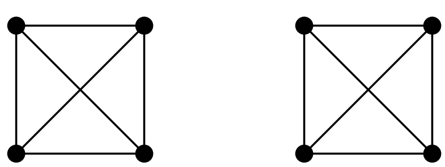
- Linking two centers can decrease welfare.

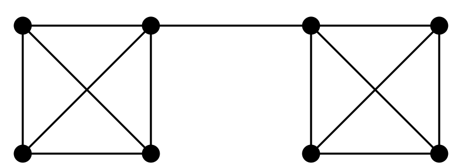
Bridges between Communities

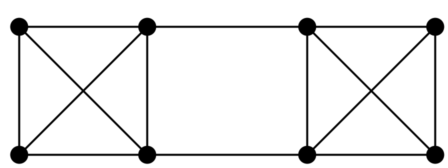
- Divide population into two communities, in which all agents are linked to each other.

- Some - but not all - agents are linked to agents in other community: *bridges* and *bridge agents*

- Members of this family of graphs described by β – the number of bridges.



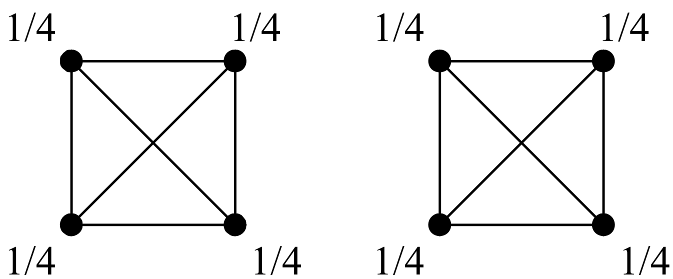


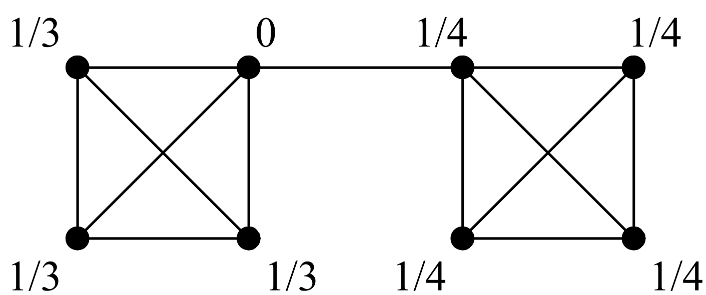


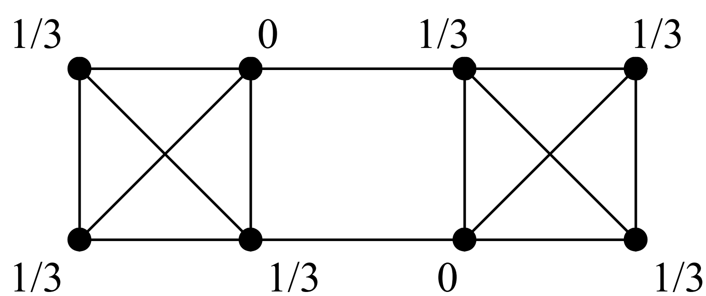
- Represents isolated villages, research units within firms....

Results:

- In equilibrium, in each community, overall experimentation is $e*$.
- For any two bridge agents, one does no experiment.
- Hence, across equilibria, average cost of non-bridge agents increases in β.







- Result counters conventional sociological wisdom about bridges – bridges help community by transmitting information.
- Here, bridge agents take advantage of other sources of information to reduce their effort.
- This reduction harms others in their community.
- This also illustrates our messages.

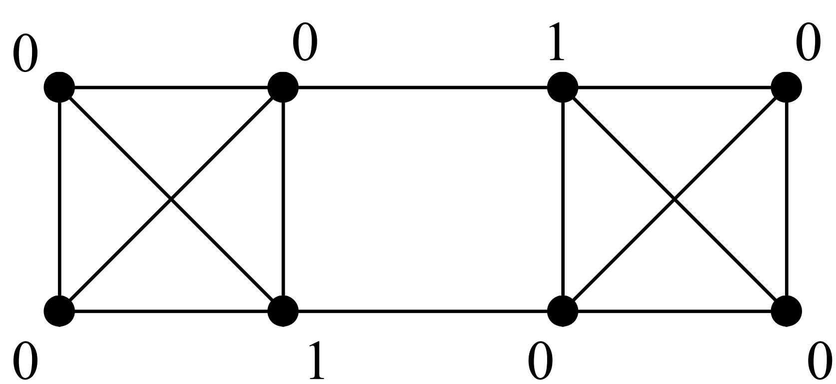
  → Better connected individuals experiment less, on average.

  → Equilibrium profiles become more inequal, on average, as β increases.

- Yet, on aggregate, welfare increases in the number of separate bridges – bridges that uniquely link agents across communities.

- Information premium for bridge agents who do not experiment.

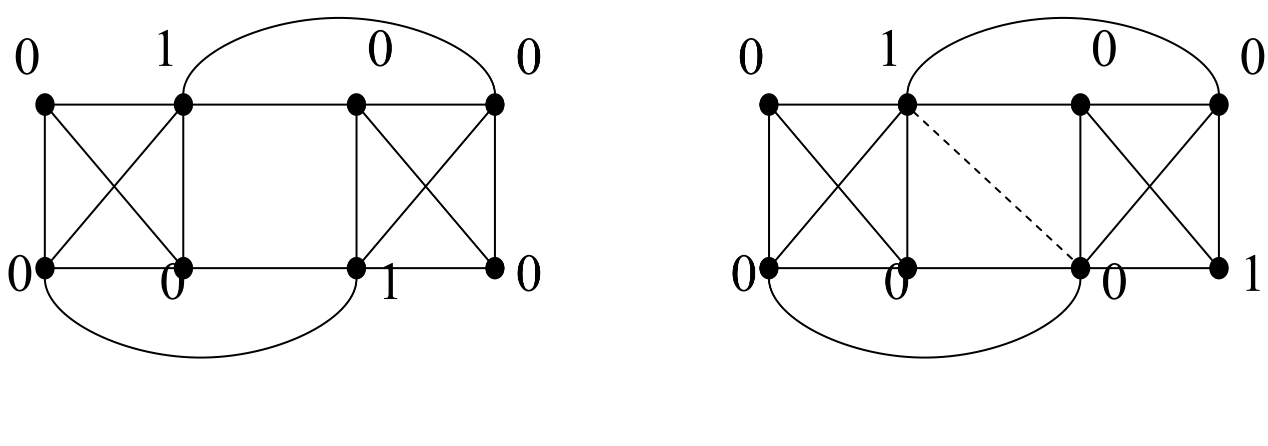
- In second-best profiles, effort is concentrated on bridge agents.



→ Welfare is higher when experimentation is done by well-connected agents.

Negative Effect of New Non-Separate Bridge

- While separate bridges increase welfare, non-separate bridges can reduce welfare.
- Consider a structure where some agents already have links to the other community.
- In second-best profile, these agents experiment.
- Adding a link between them reduces their incentive to search and can lower welfare.



Conclusion

- Paper studies the first model of strategic experimentation in networks.

- Analysis yields several insights including:

    - Social networks can determine the generation of new information.

    - Network characteristics affect overall and individual level of experimentation.

    - Networks structurally lead to specialization and effort inequality.

    - Specialization can lead to more overall experimentation and higher welfare.

    - Well-connected individuals tend to experiment less, although their effort benefits many others.

- New links reduce the incentives to experiment. Thus, might be better to have holes in a network.

- Much future work:
    - Results and predictions could be investigated empirically.
    - Information diffusion.
    - Strategic link formation.